



The Mathematicall Praeface to the Elements of Geometrie

JOHN DEE

Plain English Translation

MOUSEIA



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Translated by Michael Berardi

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THIS EDITION

Modern machine-assisted Plain English edition by Mouseia, from Dee's 1570 Praeface. Latin quotations and titles are translated into English, using Dee's own English glosses where provided. Printer's type-size markup is normalized; marginal notes are kept as asides. Billingsley's separate address is omitted. Uncertain readings are marked.

TRANSLATOR

Michael Berardi

THE WORK

Written 1570 · Treatise on the mathematical sciences

SOURCE

Original text from gutenberg.org.

GAPS AND UNCERTAINTIES

The 1570 printing's oversized fold-out Groundplat is reproduced in the source transcription as a plain list; its arrangement is the transcriber's. Greek words and quotations are kept as printed. Latin quotations are not kept in the English editions; the astronomical term *primum mobile* is kept as a conventional technical term, and *Geometria* is kept where Dee discusses the word itself. Dee's printed Latin has irregular forms, so a few readings are uncertain: the name printed *Iamne* in the Pliny sentence is read as *Jannes*, and the title *Ars Sintrillia* attributed to *Artephius* is kept as printed because its meaning and source are unclear.

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To the Lovers of Truth and Students of Noble Sciences



To all who love truth without pretense and who steadily study the noble sciences, John Dee of London warmly wishes grace from heaven and the greatest success in all their honorable work and exercises.

Divine Plato, the great teacher of many worthy philosophers, and the constant affirmer and powerful persuader of the One, the Good, and Being, was visited many times in his school and academy, besides his ordinary students, by a certain kind of men, allured by the noble fame of Plato and the great praise of his deep and profitable teaching. But when such hearers, after listening to him a long time, perceived that the point of his discourses came to this conclusion, that the One, the Good, and Being are spiritual, infinite, eternal, omnipotent, etc., with nothing alleged or explained about worldly goods, about worldly dignity, about health, strength or vigor of body, nor about the means by which a wonderful, sensible and bodily bliss and happiness hereafter might be attained—immediately those hearers were discouraged, their opinion of Plato completely changed, his doctrine was despised by them, and his school was visited by them no more. His student Aristotle, considering this closely, found the cause to be “that they had no general forewarning and information about where his teaching tended.” For so they could have had reason either to stay away from his school (if they then disliked his scope and purpose) or to con-

tinue in it steadily, to their full satisfaction, if such a final scope and intent had been what they wanted. So Aristotle, from then on, always used to warn his own students and hearers briefly, “both of what subject matter, and also to what end, he took in hand to speak or teach.” While I consider the different approaches of these two excellent philosophers (and I am most sure both that Plato could very well have taught otherwise, and that Aristotle could boldly have dealt with his hearers in the same way Plato did), I am in no small anguish of perplexity: because what I dislike is easiest for me to perform (and to have Plato as my example), while what I know to be most commendable, and (in this first bringing of the mathematical arts into common handling) most necessary, is full of great difficulty and various dangers. Yet I do not think it fitting, for so strange a matter as is now meant to be published, and to so strange an audience, to put it out bluntly at first with no special preface; nor, imitating Aristotle, can I well hope that, according to the fullness and dignity of the mathematical state, I am able either to set out plainly the material bounds, or to state precisely the chief purposes and most wonderful applications of it. And though I am sure that those who shrank from Plato’s school, after they had perceived his final conclusion, would in these things have been his most diligent hearers (so infinitely might their desires be satisfied in the end and at length by our mathematical arts), yet by this preface and forewarning all such may be drawn here sooner, to their great benefit; and also the fully trained Pythagorean and Platonian student, and the steadfast profound philosopher, may with more ease and speed gather from it both wax and honey, like the bee.

The intent of this preface.

Seeing that I have good reason (for the reasons given, and further in respect of my general mathematical art) to use “a certain forewarning and preface, whose content shall be this: the mighty, most pleasant and fruitful Mathematical Tree, with its main arms and its second grafted branches—what each one of them is, and also what benefit in general may be looked for, from graft as much as from stock. And since this undertaking is so great that, up to our time, it has never (to my knowledge) been achieved by anyone; and since it is also very hard in these gloomy days to win proper and general credit for such

rare and strange arts,” still, if you will lend me your thankful mind a while, for my sincere effort to meet your honorable expectation, and apply your eye or ear attentively to whatever my pen can deliver quickly this time, perhaps at once, as a first greeting, you will find this preface a lesson long enough. And either you will be made much readier by it for a second one, or shortly become quite able yourselves, from the lion’s claw, to judge its royal symmetry and further properties. So now, my good friends and countrymen, turn your eyes and bend your minds to that doctrine which, for our present purpose, my modest talent is able to yield you.

All things that are, and have being, fall under a general threefold difference. For either they are judged supernatural, natural, or of a third being. Supernatural things are immaterial, simple, indivisible, incorruptible, and unchangeable. Natural things are material, compound, divisible, corruptible, and changeable. Supernatural things are grasped by the mind alone; natural things can be perceived by the outward senses. In natural things probability and conjecture have a place; but in supernatural things the highest demonstration and the most certain knowledge is to be had. By these properties and comparisons of the two, the state, condition, nature and property of those things we earlier called of a third being can be more easily described, which by a special name are also called Mathematical Things. For these, being in a manner middle between supernatural and natural things, are not so complete and excellent as supernatural things, nor so base and coarse as natural things; but they are immaterial things, and yet can be signified to some degree by material things. And though their particular images can be assembled and divided by art, yet the general forms remain constant, unchangeable, untransformable, and incorruptible. They can never be perceived or judged by the senses. Nor yet for all that are they first conceived in the royal mind of man. But rising above the imperfection of conjecture, guesswork and opinion, and falling short of high intellectual understanding, they are the mediating fruit of Dianoetic reasoning, resting in perfect imagination. These mathematical things have a wonderful neutrality and a strange participation between supernatural, immortal, intellectual, simple and indivisible things, and natural, mortal, sensible, compound and divisible

things. Probability and proof from the senses may well serve in natural things, and that is commendable; in mathematical reasoning a probable argument is not regarded at all, nor is the testimony of the senses credited at all; but only a perfect demonstration of certain, necessary and invincible truths, concluded universally and necessarily, is allowed as sufficient for “an argument exactly and purely mathematical.”

Note that the word unit is used here to express the Greek monas, and not unity, as we have all commonly used until now.

Mathematical things fall into two main kinds: number and magnitude.

Number.

We define number as a certain mathematical sum of units. A unit is an indivisible mathematical thing; by partaking in some likeness of its property, anything that exists, or is counted as one, can reasonably be called one. We count a unit to be a mathematical thing, though it is no number and is also indivisible, because number consists of units materially, and number is itself chiefly a mathematical thing.

Magnitude.

Magnitude is a mathematical thing; by partaking in some likeness of its nature, anything is judged long, broad, or thick. “A thick magnitude we call a solid, or a body. Any magnitude that is solid or thick is also broad and long. A broad magnitude we call a surface or a plane. Every plane magnitude also has length. A long magnitude we call a line. A line is neither thick nor broad, but only long. Every definite line has two ends:

A point.

The ends of a line are called points. A point is an indivisible mathematical thing that may have a certain definite position.” If a point moves from a definite position, the way in which it moved is also a line, produced mathematically; after which, according to the ancient mathematicians,

A line.

A line is called the race or course of a point. We define a point as a mathematical thing, though it is no magnitude and is indivisible, because it is the proper end and bound of a line, which is a true magnitude.

Magnitude.

And we may define magnitude as that mathematical thing which is forever divisible into divisible parts, long, broad or thick. So though a point is no magnitude, yet in a limiting sense we count it a mathematical thing (as I said), because it is properly the end and bound of a line. Neither number nor magnitude has any materiality.

Of Number and Magnitude



First we will consider number, and the mathematical science proper to it, called Arithmetic; and afterward of magnitude, and its science, called geometry. But that name does not satisfy me; a word or two about that will come later. How immaterial and free from all matter number is, who does not perceive it? Yes, who does not marvel at it? For neither pure element, nor Aristotle's quintessence, can serve as the proper matter of number. Nor will the purity and simplicity of a Spiritual or Angelic Substance be found proper enough for it. And therefore the great and godly philosopher Anitius Boetius said: All things (which from the very first original being of things have been framed and made) appear to be formed by the reason of Numbers. For this was the principal example or pattern in the mind of the Creator. What a comfort, what a rapture, to deal with a Science whose Subject is so ancient, so pure, so excellent, so far above all creatures, and so used by the almighty and incomprehensible wisdom of the Creator in the distinct creation of all creatures: in all their distinct parts, properties, natures, and virtues, brought by order and the most absolute number, from Nothing, to the form of their being and state. Therefore, learning the property of Numbers by every possible means (to the perfection of the Science), we may both wind and draw ourselves into the inward and deep search and view of all creatures' distinct virtues, natures, properties, and Forms; and also further arise, climb, ascend and mount up (with speculative wings) in spirit, to see in the mirror of creation the Form of Forms, the model Number of

all things numerable: both visible and invisible, mortal and immortal, bodily and spiritual. Part of this profound and divine Science the prophet Joachim had attained: by Formal, Natural and Rational Numbers, foreseeing, concluding and foretelling great particular events long before they came. His books still remaining are good proof of this; and the noble Earl of Mirandula (besides that) is a sufficient witness that Joachim, in his prophecies, proceeded by no other way than by Formal Numbers. And this Earl himself, in Rome,

In the year 1488.

He set up 900 conclusions in all kinds of sciences, openly to be disputed, and among the rest, in his Mathematical Conclusions (in the eleventh Conclusion) he has in Latin this English sentence: "By Numbers, a way is had to the searching out and understanding of every thing able to be known." To verify which Conclusion, I promise to answer the 74 questions written below by the way of Numbers. Which Conclusions I omit here to repeat, both to avoid needless length and because the works of Ioannes Picus are commonly available. But in any case I would wish that those Conclusions were read carefully and understood by such as are earnest observers and considerers of the constant law of numbers, which is planted in natural and supernatural things, and is prescribed to all creatures to be kept inviolably. For so, besides many other things to be noted in those Conclusions, it would appear how sincerely, and within my bounds, I disclose the wonderful mysteries to be attained by numbers.

From my earlier words it is easy to gather that Number has a threefold state: one in the Creator, another in every Creature (in respect of its complete constitution), and the third in Spiritual and Angelic Minds and in the Soul of man. In the first and third state, Number is called Number Numbering. But in all Creatures, otherwise, Number is called Number Numbered. And in our Soul, Number holds such sway and has such an affinity with it that some of the old Philosophers taught that the human Soul is a Number moving itself. And indeed, in us, though it is only an accident, yet it is such an accident that before all Creatures it had perfect being in the Creator, eternally. Number Numbering, then, is the discretion that discerns and distinguishes things. But

in God the Creator this discretion, in the beginning, produced all things orderly and distinctly. For his Numbering then was his Creating of all things. And his continual Numbering of all things is their conservation in being; and wherever and whenever he will lack a Unit, there and then that particular thing will be uncreated. Here I stop. But our separating, distinguishing, and Numbering creates nothing; of Multitude considered it makes a certain and distinct determination. And although these things are weighty truths of great importance, yet (by the infinite goodness of the Almighty Ternary) Artificial Methods and easy ways are made by which the zealous Philosopher may draw near to this river-rich Ida, this Mountain of Contemplation, and more than Contemplation. And also, though Number is a thing so immaterial, so divine, and eternal, yet by degrees, little by little, stretching forth and applying some likeness of it—first to Spiritual things; then bringing it lower to things sensibly perceived, such as a momentary sound repeated; then to the smallest things that can be seen and numbered; and at length (most coarsely) to a multitude of any bodily things seen or felt—so from these coarse and sensible things we are trained to learn a certain Image or likeness of numbers, and to use Art in them for our pleasure and profit. So coarse is our dealing and dull our apprehension, while mortal Sense rules the commonwealth of our little world. By this we say, three lions are three, or a Ternary. Three eagles are three, or a Ternary.

Note this.

Such Ternaries are each the Union, knot, and Uniformity of three separate and distinct Units. That is, in each Ternary we can three times separately point out and show a part: One, One, and One. Whereas in Numbering we say One, two, Three. But how far these visible Ones differ from our Indivisible Units (principally considered in pure arithmetic) no one is ignorant. Yet from these coarse and material things we may be led upward, degree by degree, training our crude imagination toward the conceiving of Numbers absolutely (supposing no created thing, bodily or spiritual, to support, contain, or represent those Numbers imagined), that at length we may be able to find the number of our own name, gloriously exemplified and registered in the book of the most blessed and eternal Trinity.

But further understand that ordinary practitioners consider Numbers in various other ways, and extend the name further than to Numbers whose least part is a Unit. For the common reckoner, master of accounts, or arithmetician, in his use of Numbers, imagines parts smaller than a Unit and calls them Fractions. From a Unit he makes a half, and notes it thus, $\frac{1}{2}$, and so of other (infinitely various) parts of a Unit. And further, he has Fractions of Fractions, etc.

Arithmetic, 1



And, since Addition, Subtraction, Multiplication, Division and Extraction of Roots are the chief and sufficient parts of arithmetic: which is the science that demonstrates the properties of numbers, and all the operations to be performed in numbers:

“How often, therefore, these five different sorts of operations differ, for the most part in their execution, from the five operations of the same general property and name that are practicable in our whole numbers, so often (for a clearer account) we commonly count and name it another kind of arithmetic.” And by this reason:

1 The study, teaching, and working of whole numbers only, where no part smaller than a unit is allowed, is named (as it were) an arithmetic by itself. And so with the arithmetic of fractions.

2 In the same way, the necessary, wonderful, and secret doctrine of proportion and proportionality has gained for itself its own particular way of handling and working. So it may seem another form of arithmetic.

3 Also, astronomers, for speed and more convenient calculation, have devised a particular way of ordering numbers for their circular motions: by sixties and sixtieths, by signs, degrees, and minutes, etc. This is commonly called the arithmetic of astronomical or physical fractions. I have briefly noted this by the name Circular Arithmetic, because it is also used in circles that are not astronomical, etc.

4 Practice has led numbers further, and shaped them to take on the property of magnitudes, which is incommensurability and irrationality. (For in pure arithmetic, a unit is the common measure of all numbers.) Here numbers have become like lines, planes, and solids: sometimes rational, sometimes irrational. They have their own proper characters (as $\sqrt[2]{}$, $\sqrt[3]{}$, and so on for others, which is to signify square root, cube root, and so forth) and their own proper forms in the five principal parts. For that reason, the practitioner counts this a different arithmetic from the others. Practice brings in here various combinations of numbers: sometimes two, three, four (or more) radical numbers, joined in different ways by the signs of more and less, as thus $\sqrt[2]{12} \sqrt[3]{15}$, or thus $\sqrt[4]{19} \sqrt[3]{12} - \sqrt[2]{2}$, etc. And sometimes with whole numbers, or fractions of whole numbers, among them, as $20 \sqrt[2]{24}$, $\sqrt[3]{16} 33 - \sqrt[2]{10}$, $\sqrt[4]{44} 12\frac{1}{4} \sqrt[3]{9}$. And so the variety may be infinite. After this, both the one and the other have fractions attached, and so this arithmetic is greatly enlarged by the different showing and use of compositions and mixtures. Consider how I, being eager to free the student from error and quibbling, give this practice the name arithmetic of radical numbers, not of irrational or surd numbers. Those are at times rational, even though they have the sign of a root before them. The arithmetic of whole numbers, being the most usual, would say they have no such root, and so would count them as surd numbers. But that, spoken generally, is untrue, as Euclid's tenth book can teach you. So to call them, generally, radical numbers, because of the sign $\sqrt{}$ prefixed, is a sure way and a sufficient general distinction from every other ordering and use of numbers. And yet, beyond all this, consider the infinite desire for knowledge and the incredible power of human search and capacity: how together they have gone further (by mixing theory and practice) and have found and reached the very highest perfection, almost, of the practical use of numbers. This can be well seen in the great arithmetical art of equation, commonly called the Rule of Coss, or algebra. The Latins called it the rule of the thing and its value. A fitting name, covering the first and last points of the work. And the common names in Italian, French, and Spanish depend, in naming it, on the meaning of the Latin word *Res*, a thing, unless they use the name algebra. And in this there are commonly two errors.

One is the error of those who think it was Geber's invention; the other is the error of those who call it algebra. First, though Geber, for his great skill in numbers, geometry, astronomy, and other marvelous arts, might have seemed able to have first devised the said rule, and though the name carries with it a very close likeness to Geber's name, yet it is true that a Greek philosopher and mathematician named Diophantus, before Geber's time, wrote 13 books about it (of which six are still in existence, and I had them for use,

In 1550.

from the famous mathematician and my great friend Petrus Montaneus.) And secondly, the very name is Algiebar, not algebra, as the Arab Auicenna can prove, who has these exact words, translated into Latin by Andreas Alpago, most skilled in the Arabic tongue: the science of working Algiebar and Almachabel, that is, the science of finding an unknown number by adding a number, and by division, and equation. Here you have the name, and also the principal parts of the rule, touched on. To call it the rule or art of equation signifies the middle part and the state of the rule. This rule has its own particular characters:

5 and the principal parts of arithmetic belonging to it differ from the other arithmetical operations. This arithmetic has simple, compound, and mixed numbers, and fractions accordingly. This rule and the arithmetic of Algiebar are so profound and so general, and so (in a manner) contain the whole power of the practical application of numbers, that the human mind can deal with nothing more profitable in numbers, nor find anything better suited to the soul's divine power (in human studies, business, or exercises) to be tested in. Perhaps you have been expecting, long before now, some particular proof or clear testimony of the use, profit, and benefit of ordinary arithmetic in the common life and trade of people. To that, then, I will now turn. But I take great care here, for fear that the length of several proofs make you think that I either doubted your zealous mind in the school of virtue, or distrusted your able wits to guess much more. A proof, then: four, five, or six such proofs I will bring, as any reasonable person may be persuaded by them to love and honor, yes, learn and practice the excellent science of arithmetic.

And first, who closer at hand could be a better witness of the fruit received from arithmetic than all kinds of merchants? Though not all of them equally need it or use it. How could they do without the use and help of the rule called the Golden Rule, simple and compound, both forward and backward? How could they do without arithmetic's help in the rules of partnership, either without time or with time, and between the merchant and his agent? Would they gladly do without the rules of bartering in goods alone, or part in goods and part in money? Our merchant venturers and travelers overseas, how could they order their dealings justly and without loss, unless certain and general rules for the exchange of money and re-exchange were devised for their use? In how many different cases does the rule of alligation conclude for them such precise truths as they could not otherwise know, either by natural wit or by other experience? And, to make an end with the merchant, how wide-ranging and wonderful is the rule of false positions, especially as it has now been enlarged by two excellent mathematicians (of my close acquaintance in their lifetimes)? I mean Gemma Frisius and Simon Iacob. Who can briefly sum up the general and capital rules? And who can imagine the myriads of different cases and particular examples, in real earnest, continually worked out, tried, and settled by the forenamed rules alone? How many other arithmetical practices are commonly in merchants' hands and knowledge, they themselves can testify at length.

The mintmaster and the goldsmith, in their mixing of metals, whether of different kinds or different values, how exactly are they, or may they be, directed and wonderfully served, if arithmetic is their guide? And the honorable physicians will gladly confess that they owe much to the science of arithmetic, and in several ways. But chiefly in their art of graduation and in compound medicines. And though Galenus, Auerrois, Arnoldus, Lullus, and others have published their positions, both in the quantities of the degrees above temperament and in the rules that conclude the newly resulting form, yet a more precise, convenient, and easy method exists, invented by a countryman of ours

(over 200 years ago) invented. And since I am uncertain who has it, or when that little Latin treatise (as the author wrote it) will come to be printed: (both to declare the desire I have to please my country, where I can, and also as very

good proof of the use of numbers in this most subtle and fruitful philosophical conclusion,) I intend in the meantime, very briefly and with my further help, to communicate its essence to you.

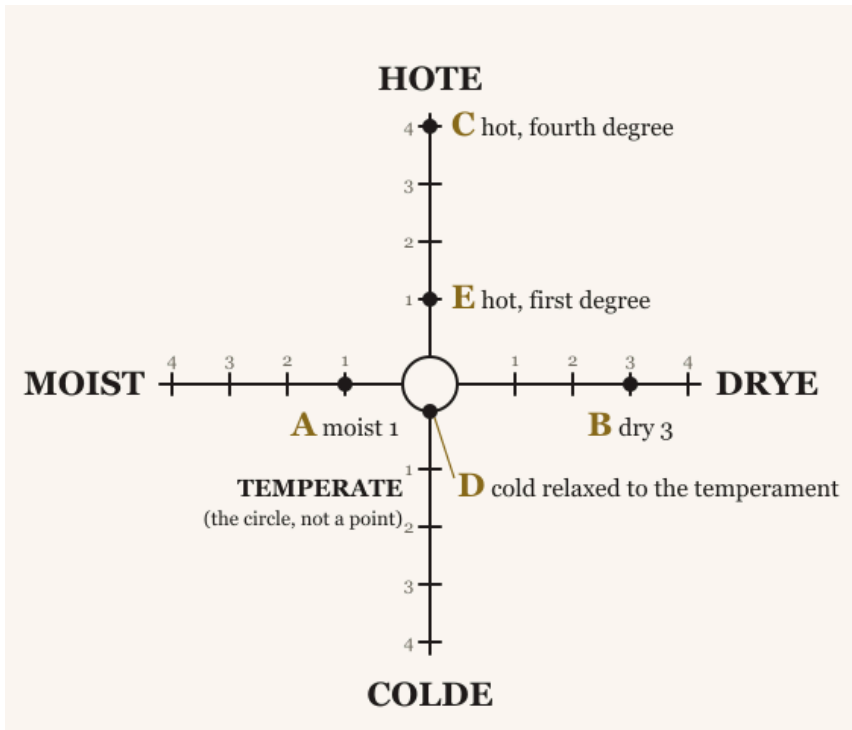
First draw a circle whose diameter is one inch. Divide the circumference into four equal parts. From the center, through those 4 points, extend 4 straight lines, each $4\frac{1}{2}$ inches long, or as long as you like, more than 4, outside the circumference of the circle: so that they are 4 inches long (at least) outside the circle. Make clear marks at the end of every inch. If you like, you may subdivide the inches again into 10 or 12 smaller equal parts. At the ends of the lines, write the names of the 4 principal elemental qualities: hot and cold, one opposite the other, and likewise moist and dry, one opposite the other. And in the circle write Temperate. This temperature has a good range, as appears in a person's complexion. And therefore we have allowed it the said circle, and not a mathematical or physical point.

Take some of Lullus's counsel in his book *On the Fifth Essence*.

Now, when you have two mixable things whose degrees are truly known, they must be either of one quantity and weight, or of different ones. If they are of one quantity and weight, whether their forms are contrary qualities, or of one kind (but of different intentions and degrees), or a temperate and a contrary, the form resulting from their mixture is in the middle between the degrees of the mixed forms. For example, let A be moist in the first degree, and B dry in the third degree. Add 1 and 3, which makes 4. The half or middle of 4 is 2. This 2 is the middle, equally distant from A and B.

Note.

For the temperament counts as none. For it you must put a zero, if at any time it is in the mixture.



The four elemental qualities and their degrees

Counting from B, 2 degrees toward A, you find it to be dry in the first degree. So is the form resulting from the mixture of A and B in our example. I will give you another example. Suppose you have two things, C and D: and the heat of C in the 4th degree, and the cold of D so weak that it reaches the temperament. For C you take 4, and for D you take a zero. Added to 4, that yields only 4. The middle, or half, of it is 2. Therefore the form resulting from C and D is hot in the second degree, because 2 degrees, counted from C toward D, end exactly in the 2nd degree of heat. I will also give an example of the third manner, which is this:

I have a liquid medicine whose quality of heat is raised to the 4th degree, as C was in the example above. And I have another liquid medicine whose quality is heat in the first degree. Of each of these I mixed an equal quantity. Subtract

the lesser from the greater here, and divide the remainder into two equal parts. One of those two parts, either added to the lesser or subtracted from the higher degree, produces the degree of the form resulting from this mixture of C and E. Thus, if from 4 you take away 1, 3 remains. The half of 3 is $1\frac{1}{2}$. Add this $1\frac{1}{2}$ to 1, and you have $2\frac{1}{2}$. Or subtract this $1\frac{1}{2}$ from 4, and you likewise have $2\frac{1}{2}$ remaining. This shows the resulting form to be heat in the middle of the third degree.

The second rule.

“But if the quantities of two things mixed together are different, and the intensities (of their mixable forms) are in different degrees and heights. (Whether those forms are of one kind, or of contrary kinds, or a temperate and a contrary, what proportion the lesser quantity has to the greater, the same proportion shall the difference between the degree of the resulting form and the degree of the greater quantity of the mixable thing have to the difference between that same degree of the resulting form and the degree of the lesser quantity.) As for example. Let two pounds of liquor be given, hot in the 4th degree, and one pound of liquor be given, hot in the third degree.” I would gladly know the form resulting in the mixture of these two liquors. Set down your numbers in order, like this.

£. 2. Hot. 4.

£. 1. Hot. 3.

Arithmetic, 2



By the rule of algebra I have devised a very easy, brief and general way of working in this case. First, let us suppose that the middle resulting form is $1x$, as that rule teaches. For, by the rule given here: as the weight of 1 is to 2, so is the difference between 4 (the degree of the greater quantity) and $1x$ to the difference between $1x$ and 3 (the degree of the thing in lesser quantity; and all along, $1x$ always sits at a certain middle between the two heights or degrees). For the first difference I set $4 - 1x$; for the second, I set $1x - 3$. And now again I say: as 1 is to 2, so is $4 - 1x$ to $1x - 3$. Of these four proportional numbers, then, the first and the fourth multiplied together make as much as the second and the third multiplied together. Let the multiplications be made accordingly. From the first and the fourth we have $1x - 3$, and from the second and the third, $8 - 2x$. So our equation lies between $1x - 3$ and $8 - 2x$. It can be reduced by the art of algebra: here, adding 3 to each side gives the equation, thus, $1x = 11 - 2x$. And to reduce it again: add $2x$ to each side. Then you have $3x$ equal to 11, written $3x = 11$. Dividing 11 by 3, the quotient is $3\frac{2}{3}$: the value of our $1x$, the Coss, or Thing, first supposed. That is the height, or intensity, of the resulting form: it is heat at two thirds of the fourth degree. And here I set out the work itself in conclusion, like this. The proof is easy. Subtract 3 from $3\frac{2}{3}$ and $\frac{2}{3}$ remains. Subtract the same height of the resulting form (which is $3\frac{2}{3}$) from 4, and $\frac{1}{3}$ remains. You see that $\frac{2}{3}$ is double $\frac{1}{3}$, as 2 parts are double 1 part. So it should be, by the

rule given here. Note: as you added 3 to each side of the equation, so if you had first added $2x$ to each side, it would stand as $3x - 3 = 8$. And now, adding 3 to each side, you have (as before) $3x = 11$.

Two parts, heat 4, $\frac{1}{3}$; the form resulting, $3\frac{2}{3}$; one part, heat 3, $\frac{2}{3}$.

Although I speak here of only two mixable things, and most commonly more than three, four, five or six (etc.) are to be mixed (and reduced into one compound, with the resulting form of them doing the job), yet these rules are enough, if they are repeated and applied in turn.

Start with any two; then take the resulting form and another; and so on. For the last operation gives the resulting form of all of them. I need say nothing here about what the mixture (supposed here) is. Common philosophy has defined it, saying: Mixture is the union of miscible things that have been altered and joined through their smallest parts. Every word in the definition is of great importance. Nor do I need to spend any time showing how the other way of distributing degrees agrees with these rules. Neither do I need to explain here the further use of the Cross of Graduation (described before) to those who can take in what I have already said. Nor do I need to specify with examples the many varieties that the two general rules above can order. The clever and the studious have enough here. Those who cannot reach this without live teaching, and in more detail, would need a longer discussion than belongs in this place. And others (perhaps) will turn up their noses at this little, and would be ungrateful for much more. So I conclude, and I wish those with modest and earnest philosophical minds to praise God highly for this, and to marvel that the deepest and subtlest point concerning the mixing of forms and natural qualities is so matched and married to the simplest, easiest and shortest way of the noble rule of algebra. Who, then, can stay unpersuaded from loving, approving and honoring the excellent science of arithmetic? For here you can see that the little finger of arithmetic has more power and contrivance than the wits of a hundred thousand ordinary people can perform or truly conclude without its help.

Now we will go further, and by the wise and brave captain learn what help he has from the rules of arithmetic in one of the arts belonging to him, which the Greeks call

Τακτικῇ.

Τακτικῇ. “That is, the skill of ordering soldiers in battle array in the best way for all purposes.” This art depends so much on the use of numbers and on mathematics that Ælianus (the best writer on it), in his work for the Emperor Hadrianus, thinks his book, by his perfection in mathematics (which was greater than others before him had), surpasses all other excellent works on that art written up to his day. For on it had written Æneas; Cyneas of Thessaly; Pyrrhus Epirota and Alexander his son; Clearchus; Pausanias; Euangelus; Polybius, familiar friend of Scipio; Eupolemus; Iphicrates; Possidonius; and very many other worthy captains, philosophers and princes of immortal fame and memory, whose fairest flower in their garland (in this feat) was arithmetic, and a little knowledge of geometric figures. But in many other cases arithmetic stands the captain in good stead: in apportioning provisions for the army, whether it stays at its current size, or is suddenly increased by a certain number of soldiers, for a certain time; or, by good skill, in reducing his company, so that the provisions last longer for those who remain, for a fixed period, if need requires. And so, in his other accounts, reckonings, measurements and proportions, the wise, expert and careful captain will affirm that the science of arithmetic is one of his chief counselors, directors and helpers. This was clear, by good means, to the noble, courageous, loyal and courteous John, late Earl of Warwick. He was a young gentleman thoroughly known by very few. Yet his lively valor, strength and skill in chivalrous feats and exercises, and his humility and friendliness to all men, were things the world saw openly. But what roots virtue had otherwise fastened in his breast; what rules of godly and honorable life he had set for himself; what notable vices (in some then living) he took great care to avoid; what manly virtues (in other noblemen flourishing before his eyes) he constantly aspired to; what prowess he intended and meant to achieve; and with what feats and arts he began to furnish and load himself for better service to his king and country, in peace and war: these, I say, his heroic medita-

tions, forecasts and decisions, no two people besides myself, I think, can report so perfectly and truly. And so, in conscience, I count it my part, for the honor, advancement and promotion of virtue, briefly to have put his name in the Register of Immortal Fame.

To our point. This John, by one of his acts (besides many others, both in England and France, that I noted in him), revealed his heartfelt love of the virtuous sciences and his noble aim to excel in martial prowess, when he obtained, by humble request and constant urging, the best rules (used in times past by Greeks or Romans, or used in our own time, with new stratagems devised in them) for ordering all companies, bodies and numbers of men (many or few) appointed with one kind of weapon or more, with artillery or without, on horseback or on foot; to give or take the onset; to seem many when few, and few when many; to march in battle or on a journey; and many such feats belonging to a pitched battle, skirmish or ambush:

This noble earl died in the year 1554, scarcely 24 years of age, having no children by his wife, daughter to the Duke of Somerset.

And all these lively designs, most carefully, were to be drawn on vellum parchment, with notes and special marks as the art requires; and all these rules and arithmetic descriptions, enclosed in a rich case of gold, he used to wear around his neck as his most precious jewel and his most trusted counselor. Thus arithmetic was enshrined in him in gold; of the fruit of numbers he had good hope. Now, therefore, innumerable numbers shall find, in the praise of numbers, his shrine.

Why need I (for further proof to you) ask the schoolmasters of justice for testimony of how necessary, how fruitful and how skillful a thing arithmetic is? I mean the lawyers of all sorts. Undoubtedly the civil lawyers can wonderfully show how neither the ancient Roman laws can be understood without good knowledge of the art of numbers, nor can justice (in infinite cases) be carried out without due proportion, closely considered. How justly, and with great knowledge of the art, did Papinianus lay down a law of division and allowance between man and wife after a divorce? But how Accursius, Baldus, Bartolus, Iason, Alexander, and finally Alciatus (otherwise notably well learned) jumble,

guess and err from the equity, art and intent of the lawmaker, arithmetic can detect and convict, and can clearly make the truth shine. Good Bartolus, worn out from examining and apportioning the matter, and much troubled by Accursius's gloss, burst out and said: In the whole book there is no gloss harder than this, whose account or reckoning neither the scholars nor the doctors understand, etc. What can they say of the law of Julianus beginning "If it was thus written," etc., of the testator's will justly performed between the wife, son and daughter? How can they perceive the equity of Africanius's arithmetic reckoning, where he treats of the Falcidian Law? How can they clear him from his reprovers and their maintainers, such as Ioannes, Accursius Hypolitus and Alciatus? How justly and skillfully was Africanius's reckoning made, apportioning to the sums bequeathed the contributions of each part? Namely, for the hundred received right away, $17\frac{1}{7}$; and for the hundred received after ten months, $12\frac{6}{7}$; which make the 30 that the legatees were to contribute to the heir. For what proportion 100 has to 75, the same $17\frac{1}{7}$ has to $12\frac{6}{7}$; which is sesquiquarta, that is, as 4 to 3, which make 7. Very many places in the civil law require an expert arithmetician to understand the deep judgment and just determination of the ancient Roman lawmakers. But he ought to be far more expert who can decide with equity the infinite variety of cases that do or may happen under every one of those civil laws and ordinances. From this you may easily conjecture that in canon law, and in the laws of the realm (which with us carry the chief authority), justice and equity might be greatly preferred and skillfully executed, through due skill of arithmetic and the proportions belonging to it. The worthy philosophers and prudent lawmakers (who have written many books On the Commonwealth: how the best state of commonwealths might be procured and maintained) have judged very well of justice (which is not only the base and foundation of commonwealths, but also the total perfection of all our works, words and thoughts), defining it,

Justice.

"to be that virtue by which to everyone is rendered what belongs to him." God demands this of us: to be honored as God, to be loved as a father, to be feared as a Lord and master. Our neighbor's proportion is also laid down by the

Almighty lawmaker: to do to others as we would have done to us. These proportions are necessary in justice, commendable in duty, and for commonwealths they are the life, strength, support and flourishing. Aristotle in his *Ethics* (to fetch the seed of justice and the light of direction, to use and execute the same) had to turn to the perfection and power of numbers: for arithmetic and geometric proportions. Plato in his book called *Epinomis* (which book is the treasury of all his doctrine), where his purpose is to seek a science that, when a man had it perfectly, would make him seem and so actually be wise. He, briefly discussing other sciences, finds them not able to bring it about; but of the science of numbers he says: That science, surely, which has taught mankind number, will be able to bring it about. And I think that a certain god, rather than fortune, has given us this gift for our happiness. For why should we not judge him who is the author of all good things to be also the cause of the greatest good thing, namely wisdom? There, at length, he proves wisdom to be attained by good skill in numbers. With that great testimony, and the many proofs and reasons expressed before, you may be sufficiently and fully persuaded of the perfect science of arithmetic, and come to this account: that of all sciences, next to theology, it is most divine, most pure, most ample and general, most profound, most subtle, most useful and most necessary. Its next sister is the absolute science of magnitudes; of which (by the direction and help of him whose magnitude is infinite and of us incomprehensible) I now intend so to write that, both with the multitude and with the magnitude of marvelous and fruitful truths, you (my friends and countrymen) may be stirred up and awakened to see what certain arts and sciences (for our unspeakable benefit) our heavenly Father has prepared and revealed for us by various philosophers and mathematicians.

Arithmetic, 3



Both number and magnitude have a certain original seed (as it were) of incredible property, which man is never able to declare fully. For number, a unit, and for magnitude, a point, seem to be very much alike as original causes; but the difference is nevertheless great. We defined a unit to be a mathematical thing indivisible; a point, likewise, we said to be a mathematical thing indivisible. Further, a point may have a certain fixed situation: that is, we may assign and set a point to be here, there, over there, etc. Here, notice, our unit is free, and can endure no bondage, nor be tied to any place or seat, divisible or indivisible. Again, because a point may have a situation limited to it, a certain motion (to a place and from a place) belongs to a point. But a unit cannot be imagined to have any motion. A point, by its motion, produces mathematically a line (as we said before), which is the first kind of magnitudes and the simplest; a unit cannot produce any number. A line, though it is produced by a point moved, does not consist of points; number, though it is not produced by a unit, does consist of units as a material cause. But formally,

Number.

Number is the union and unity of units. This uniting and joining is the workmanship of our mind, which, out of distinct and separate units, makes a number by the uniformity resulting from a certain multitude of units. And so every number can be given its least part, namely a unit; but of a magnitude (not even of a line) can the least part be given, because its division can be conceived

without end. All magnitude is either a line, a plane or a solid. A line, plane or solid can be perceived by no sense, nor represented exactly by hand in any way, nor produced by nature; but as number came to our perception by degrees, so we are helped by visible forms to imagine what our mathematical line is, and what our point is. Our magnitudes are so precise that one line is no broader than another, for they have no breadth; nor do our planes have any thickness; nor do our bodies have any weight, however large in dimension. We can have bodies smaller than anything art or nature can produce, and greater also than all the world can contain. Our least magnitudes can be divided into as many parts as the greatest. For a line an inch long (with us) can be divided into as many parts as the diameter of the whole world from east to west, or extended any way. What privileges, above all manual art and nature's power, have our two mathematical sciences: to exhibit and deal with things of such power, liberty, simplicity, purity and perfection; and in them to proceed so certainly, so orderly, so precisely? Excellent is that mechanical workman judged who can come nearest to representing works mathematically demonstrated.

And our two sciences, remaining pure and absolute in their proper terms and in their own matter, to have and allow only such demonstrations as are plain, certain, universal and of eternal truth?

Geometry



This science of magnitude, its properties, conditions, and appurtenances, has commonly been called geometry, now and from the beginning, by all philosophers. But truly, that name is too base and too small for a science of such dignity and breadth. And perhaps that name has been allowed to remain, by the common and secret agreement of all wise people, so that it carries a perpetual memory of the first and greatest benefit that this science showed to common people: which was when the boundaries and markers of land were lost and confused (as in Egypt every year, with the overflowing of the Nile, the greatest and longest river in the world), or when bequeathed land had to be assigned, or sold land had to be laid out, or (when disorder prevailed) commons had to be divided into separate holdings. For, on these and similar occasions, some by ignorance, some by negligence, some by fraud, and some by violence, wrongly limited, measured, encroached on, or claimed (under the pretense of a fair content and measure) those lands and grounds: great loss, unrest, murder, and war very often followed. Until, by God's mercy and human effort, the perfect science of lines, planes, and solids (like a divine judge) gave to each person what was their own. The people, pleased by this art and greatly helped by the fair measuring of their lands, and other philosophers writing rules for land measuring, between them both confirmed the name *Geometria*: that is, according to the very etymology of the word, land measuring. In this, the people knew no further use of magnitudes than in planes; and the philosophers had no fit hear-

ers or students to disclose anything further to them than flat, plain geometry. And though these philosophers knew of further use and best understood the etymology of the word, yet they applied the name Geometria generally to all sorts of magnitudes: unless, at times, Plato and Pythagoras, when they wanted to state their own teaching precisely. Then, was

(* Plato, Republic 7.)

* Geometria, with them, the study concerned with the plane. But you can see clearly enough from Euclid's Elements that our science is wider than measuring planes, and that nothing less is taught in it (on purpose) than how to measure land. So another name must be had for our mathematical science of magnitudes: one that cares nothing for clod or turf, hill or dale, earth or heaven, but is absolute Megethologia: not creeping along the ground and dazzling the eye with pole, perch, rod, or line, but "lifting the heart above the heavens by invisible lines, and

immortal beams meet with the reflections of the incomprehensible light, and so bring joy and unspeakable perfection." Of this true use of our Megethica, or Megethologia, divine Plato seemed to have good taste and judgment: and (by the name of geometry) he noted it in that way, and warned his students of it, as may clearly be seen in his seventh dialogue, of the Commonwealth. Where it is thus, and well translated: Truly (says Plato) whoever has tasted even a very little of geometry will not deny us this: that this science is of a quite contrary condition to what those who are occupied in it say about it. And it follows, of our geometry: It must be confessed (says Plato) that [geometry] is learned for the knowledge of that which always is, and not of that which, in time, both comes into being and comes to an end. etc. Geometry is the knowledge of what is everlasting. It will therefore lift up our mind to the Truth (O Gentle Sir), and by that means prepare the thought for the philosophical love of wisdom: so that we may turn or convert toward heavenly things [both mind and thought], which now, contrary to what is fitting, we cast down onto base and inferior things. etc. Commandment must therefore chiefly be given, that those who inhabit this most honorable City by no means despise geometry. For even those things [done by it] which seem, in a manner, to lie

beside the purpose of geometry are of no small importance. etc. And besides the manifold uses of geometry in matters relating to war, he adds more: a second, unintended fruit and benefit arising from geometry, saying: But also we know that, for the easier learning of all arts, it much matters whether one has any knowledge of geometry or not. etc. Let us therefore make an ordinance or decree, that this science be learned by young men in the second place. This was divine Plato's judgment, both of the intended, chief, and perfect use of geometry, and of its second, dependent, derivative benefits. And for us, Christians, there are a thousand thousand more occasions to need the help of*

(I. D. * In this, I would gladly shake off the earthly name of geometry.)

Megethologicall Contemplations: by which to train our imaginations and minds, little by little, to forsake and abandon the gross and corruptible objects of our outward senses; and to grasp, by sure demonstrative teaching, mathematical things. And by them, to be readily helped and guided to conceive, reason about, and conclude on things intellectual, spiritual, eternal, and such as concern our everlasting bliss: which otherwise (without a special privilege of illumination or revelation from heaven) no mortal person's wit can naturally reach or grasp. And truly, by my small talent from above, I am able to prove and testify that the literal text and order of our divine Law, Oracles, and Mysteries require more skill in numbers and magnitudes than expositors have commonly stated: they have rather only, at most, warned of this and shown their own lack in it. (To name any is needless, and to note the places is no part of this: but if I am properly asked, my answer is ready.) And without the literal, grammatical, mathematical, or natural truths of such places, perceived by good and certain art, no spiritual sense (proper to those places, by absolute theology) will depend on them.

"No one, therefore, can doubt that, toward the attaining of incomparable knowledge and heavenly wisdom, mathematical speculations, both of numbers and magnitudes, are means, aids, and guides: ready, certain, and necessary." From now on, in this preface of mine, I will shape my talk for Plato's runaway students: or rather, for those who can well, and also will, use their outward senses, to the glory of God, the benefit of their country, and their own private

contentment or honest advancement on this earthly stage. To them I will orderly recite, describe, and declare a great number of arts, derived from our two mathematical fountains into the fields of Nature. By which such seeds and roots as lie deep hidden in the ground of Nature are refreshed, quickened, and provoked to grow, shoot up, flower, and bear fruit, infinite and incredible. And these arts will be such as depend more on the properties of magnitudes than on number. And with good reason we may call them arts, and mathematically derivative arts: for (at this time) I define

(An Art.)

An art, to be a methodical and complete body of teaching, having plenty of sufficient and particular matter to deal with, by the allowance of the metaphysical philosopher: knowledge of which is necessary to human life. And that I count,

(A Mathematically Derivative Art.)

An art mathematically derivative, which by a mathematical demonstrative method, in numbers or magnitudes, orders and confirms its teaching, as much and as perfectly as the subject matter allows. And because I intend to use the name and property of a

(A Mechanician.)

Mechanician, otherwise than it has been used so far, I think it good (for the sake of distinction) to give you also a brief description of what I mean by it. A mechanician, or a mechanical workman, is a person whose skill is to work and finish any sensible work perfectly, without knowledge of mathematical demonstration, by the principal or derivative mathematician, demonstrated or demonstrable. I know very well that whoever invents or makes these demonstrations is generally called a speculative mechanician, which differs nothing from a mechanical mathematician. So, in view of different actions, one person may hold the names of several arts: at one time a logician, at another (when the same matter is handled otherwise) a rhetorician. Of these trifles I make little account (for now, in view of my preface): I leave them for the fine handling of subtle, exacting disputers. In other places they may command me to give a good reason; but here I will not be unreasonable.

(1.)

First, then, from the purity, absoluteness, and immateriality of principal geometry is derived that kind of geometry which is commonly counted as geometry: and it is the art of measuring perceptible magnitudes, their exact quantities and contents.

Common Geometry



This teaches measuring up close: the practitioner is at the thing being measured, and so, by correctly applying a compass, rule, square, yard, ell, perch, pole, line, gaging rod, (or such like instrument) to the length, plane, or solid being measured,

(1.)

* to learn either the length, perimeter, or linear distance; and this is called mecometry. Or

(2.)

* to learn the content of any plane surface: whether it is ground surveyed, board measured, or glass measured, or such like thing; which measuring is named embadometry.

(3.)

* Or else to understand the solidity and content of any bodily thing: as of timber and stone, or the content of pits, ponds, wells, vessels, small and great, of all shapes. Where the measuring of wine, oil, beer, or ale vessels, etc. commonly has a particular name, and is called gaging. And the general name of these solid measures is stereometry.

(2.)

Or else, this common geometry also aims to teach the practitioner how to measure things from a good distance, keeping apart from the thing measured, and by that to understand either

(1.)

* how far away a thing seen (on land or water) is from the measurer; and this may be called apomecometry:

(2.)

Or, how high or deep, above or below the level of the measurer's standing, anything seen on land or water is; called hypsometry.

(3.)

* Or, it tells the measurer how broad anything is that lies in the measurer's view, as long as it is situated on land or water; and may be called platometry. Though I make it a condition here that the thing measured be situated on land or water:

yet know for certain that the various heights of clouds, blazing stars, and the moon can (by these means) have their distances from the earth determined; and that the solidity of the blazing stars and the moon can be measured as well as their distances. But because these things are not commonly taught, nor so easily carried out by an ordinary practitioner, I prefer to count such measures as belonging to some of our other arts, which deal more purposely with things up high, rather than to this common land-measuring geometry: as in perspective and astronomy, etc.

From these feats (applied further) has sprung the feat of geodesy, or land measuring: to measure and survey land, woods, and waters more skillfully from afar. More skillfully, I say. But God knows (so far, in these realms of England and Ireland, whether through ignorance or fraud, I cannot tell in every particular)

how much wrong and injury has been done (in my time) by untrue measuring and surveying of land or woods, in any way. And this I am sure of: the value of the difference between the truth and such surveys could have kept (forever) an excellent mathematics reader in each of our two universities; allowing each (yearly) a hundred marks of lawful money of this realm, which indeed would seem to be needed here (though provided for in other ways), just as the famous University of Paris has two mathematics readers, each with two hundred French crowns a year, from the French king's magnificent generosity alone.

Now, returning again to our purpose: moreover, from the earlier geometric knowledge have grown the skills of geography, chorography, hydrography, and Stratarithmetrie.

Geography



“Geography teaches the ways in which, in various forms (as spherical, plain, or other), the situation of cities, towns, villages, forts, castles, mountains, woods, harbors, rivers, creeks, and such other things, on the outer face of the earthly globe (either in the whole, or contained in some principal member and portion of it) may be described and drawn, in proportions analogous to nature and truth, and may be represented most fitly to our view.” Of this art, how great pleasure and how many benefits come to us daily and hourly is perceived by most people. While some use them to beautify their halls, parlors, chambers, galleries, studies, or libraries; others for things past, such as battles fought, earthquakes, heavenly fires, and such occurrences as are mentioned in histories, by that to view, as it were, the place, the neighboring region, the distance from us, and such other circumstances. Some others, to view at once the large dominion of the Turk: the wide Empire of the Muscovite; and the little piece of ground where Christendom (by profession) is certainly known. Little, I say, in respect of the rest. etc. Some, either to direct their own journeys into far lands, or to understand other people’s travels. To conclude, some for one purpose and some for another like, love, get, and use maps, charts, and geographic globes. Of whose use, to speak fully, would require a book of its own.

Chorography



Chorography seems to be a lesser thing, and a twig, of geography: and yet nevertheless it is manifold in practice and very wide in use. “This teaches, by analogy, to describe a small portion or circuit of ground, with its contents: not regarding what commensuration it has to the whole, or any parcel contained outside it. But in the territory or parcel of ground which it takes in hand to describe, it leaves out (or undescribed) no notable or odd thing above the ground that is visible. And sometimes of things under ground it gives some particular mark or warning: as of metal mines, coal pits, stone quarries, etc.” Thus, a duchy, a shire, a lordship, or less, may be described distinctly. But it is marvelous pleasant and profitable to display to our eye, and measure in proportion, the plan of a city, town, fort, or palace, in true symmetry: without going close to any of them, and from out of gunshot, etc. By that the architect can furnish himself with as many patterns as he likes: to his great instruction, even in those things that are proportioned outwardly: either simply in themselves, or in relation to hills, rivers, harbors, and woods adjoining. Some also call this particular description of places topography.

Hydrography



“Hydrography delivers to our knowledge, on globe or in plain, the perfect analogous description of the ocean sea coasts, through the whole world: or in the chief and principal parts of it:” with the islands and chief particular places of danger contained within the bounds, and sea coasts described: as of quicksands, banks, pits, rocks, races, countertides, whirlpools, etc. This deals chiefly with the element of water: as geography principally took the description of the element of earth (with its appurtenances) to task. And besides this, hydrography requires a particular register of certain landmarks (where marks can be had) from the sea, well able to be sighted: at what point of the sea compass they appear, and what apparent shape, situation, and bigness they have, in respect of any dangerous place in the sea, or near to it, assigned. And on all coasts, what moon makes full sea, and what way the tides and ebbs come and go, the hydrographer ought to record. The soundings likewise, and the channels’ ways: their number and depths ordinarily at ebb and flood, the hydrographer ought to have certainly known by observation and careful measuring. And many other points belong to perfect hydrography, and to making up sailing directions, of which I need not speak here: such as describing, in any place, on globe or plain, the 32 points of the compass truly (of which scarcely four in England have right knowledge, because its lines are neither straight lines nor circles). Making a due projection of a sphere on a plain. The variation of the compass from true north: and such like matters (all of great importance) I will not speak of here:

because I may seem already to have enlarged the bounds and duty of a hydrographer much more than anyone (to this day) has noted or prescribed. Yet I am well able to prove that all these things pertain, and are also proper, to the hydrographer. The chief use and end of this art is the art of navigation: but it has other diverse uses: even to be enjoyed by those who never lose sight of land.

Stratarithmetrie



Stratarithmetrie is the skill, used in war, by which a man can set any set number or total of men into a figure like any given geometric figure, provided that figure can hold them. Two things limit it: the usual spaces allowed between soldiers, and the fact that men cannot be divided into fractions. Even so, he can arrange the given number of men into the largest figure they can fill, and report the surplus, if there is any. He can also report the next set number that, with the surplus, will make up a figure exactly like the figure assigned. The same skill works the other way. Given any army or company of men, whose figure and sides in orderly standing or array are known, he can state the exact number of men contained in that figure, or that could be contained in it in order.

He can also know this figure and its sides, whether he is near at hand or far off. That is as far as the description and the nature of Stratarithmetrie goes, enough for this time and place.

The difference between Stratarithmetrie and tactics.

“It differs from the art of tactics (drawing up battle lines), because there you need the wisdom and foresight to know what purpose you are ordering the men for, and the skilled ability to design and use, for any occasion or purpose, the fittest and most necessary order, array, and figure for your company and total of men.” By figure I mean a square, triangle, circle, oval, rectangle (the Greeks call it Eteromekes), rhombus, rhomboid, crescent, ring, serpentine, and such other geometric figures, which have been and are to be used in war for

convenience, necessity, and advantage, etc. He must have no small skill who would give a true report, or one near the truth, of the numbers and totals of foot soldiers or horsemen in the enemy's formation. To estimate from far off, down to close limits of more and less, is not common, even among those who would gladly do it.

I. D. — my friend, you will find it hard to carry out my description of this work. But by chorography you can help yourself somewhat, where the figures known by their sides and angles are not regular, and where resolving them into triangles can serve, etc. And yet you will find it strange to deal so generally with arithmetic figures, and for a battle array, since their contents differ so much from those of like geometric figures.

Captains can make good use of policy, at fitting times and in suitable places: using figures that make the greatest show of the number they have, and, taking advantage of the three usual kinds of space between foot soldiers or horsemen, taking the largest; or, when they want to seem few though they are many, doing the opposite in both figure and space. The herald, pursuivant, sergeant royal, captain, or anyone careful to come near the truth here, besides the judgment of his expert eye, his skill in tactical ordering, and the help of his geometric instrument — a ring or an astronomical staff, built conveniently for carrying and use — can wonderfully help himself with perspective glasses, in which, I trust, our descendants will prove more skillful and expert, and for greater purposes, than in these days can almost be credited as possible.

Thus I have quickly passed over the artificial feats that depend chiefly on common geometry, and that are usually and generally counted under the name of geometry.

The Mixed Sciences and Perspective



But there are very many other methodical arts. Departing from the purity, simplicity, and immateriality of our principal science of magnitudes, they still make great use of the help, direction, and method of that principal science, and they have their own names, distinct both from the science of geometry, from which they are derived, and from one another: such as perspective, astronomy, music, Cosmography, astrology, statics, anthropography, Trochilike, Helicosophy, Pneumatithmie, Menadrie, Hypogeiodie, Hydragogie, horometry, painting, architecture, navigation, Thaumaturgike, and Archemastrie. I think it necessary to describe these in order, and also to touch on some of their useful purposes, and so to make this Preface a little sweet, pleasant nosegay for you: to comfort your spirits when you are almost out of courage and in despair, through a brutish way of thinking, imagining that geometry had only served to build a house, or a fine bridge, or the roof of Westminster Hall, or some clever little device or engine belonging to a carpenter or a joiner, etc. The truth is far different from what the world has commonly judged, by word and deed, to this day, and good proof of that will be made.

Among these arts, perspective rightly ought to come first, before any perfect knowledge of astronomical appearances can be attained. And because of the special place of light, being the first of God's creations, and because the eye is

the light of our body, our most powerful sense, and our most skillful and geometric organ, we will begin with perspective. Perspective is a mathematical art that demonstrates the manner and properties of all radiations: direct, broken, and reflected. This description is brief, but it reaches as far as the world is wide. It concerns all creatures, and all actions and passions performed by the emission of rays. By rays, or natural lines, I mean here not only rays of light or color — though these give the eye the appearance, witness, and proof on which to ground the art — but also the certain and determined active radial emissions of other forms, both substantial and accidental. By this art, leaving aside the highest points, we can use our eyes and the light with greater pleasure and better judgment: both of things seen in light and of other things, which work and produce their effects by the same order of light's radiations. We should be ashamed not to know the cause of the many ways our eye is deceived and misled: how the eye takes a round globe or sphere, seen from far off, to be a flat plain circle, and likewise judges a plain square to be round; supposes parallel walls to draw together from far away; and takes the roof and floor, the one to bend downward and the other to rise upward, when they are a little distance from you. Again, of things moving at the same speed, it thinks the nearer moves faster and the farther much slower. Or, of two things, one of which moves incomparably faster than the other, it judges the slower to move very fast and the other to stand still: what an error of our eye! Of the rainbow — its colors, the order of the colors, its size, its place and height, etc. — is it not pleasant and necessary to know the demonstrative causes? Of two or three suns appearing, of blazing stars, and such like things, brought to pass by natural causes, and yet at the same time signifying matters beyond them: is it not useful for people to know their true and natural cause and occasion? In fact, is it not greatly beneath the dignity of human nature to be so tricked and misled by things at hand, before one's eyes? Such as a peacock's tail and a dove's neck; or a whole oar held in water, seeming broken. Things far off seeming near, and near seeming far off. Small things seeming great, and great things small. One man seeming like an army. Or a person terribly afraid of his own shadow. So much so

that, if you are alone near a certain mirror and try to thrust at it with a dagger or sword, you will suddenly be moved to give back, in a manner, by reason of an Image,

A marvelous mirror.

appearing in the air between you and the mirror, with a like hand, sword or dagger, and with like quickness, thrusting at your very eye, just as you do at the mirror. This is strange to hear of, but more marvelous to see than these words of mine can express. And yet by optical demonstration the order and cause of it are just as certain as the effect that follows. I dare take it upon myself even further, to satisfy the noble courage that longs ardently for the wisdom of natural causes: to let him understand that in London he can have proof of what I have said here with his own eyes. A gentleman, who, for his good service to his country, is famous and honorable:

and for skill in the mathematical sciences and languages, is the extraordinary man of this land, etc. He himself is able, and I am sure will very willingly, let the mirror and the proof be seen, and so I ask it of him here: for the increase of wisdom among the honorable, for stopping the mouths of the malicious, and for repressing the arrogance of the ignorant. You can easily guess what I mean. This art of perspective is of such excellence, and can lead to the certifying and carrying out of such things as no one would easily believe without actual proof. I say nothing of natural philosophy, which without perspective cannot be fully understood nor perfectly attained. Nor of astronomy, which without perspective cannot be well grounded; nor of astrology, naturally verified and confirmed. That part of it which deals with glasses — where glass is a general name in this art for anything from which a ray rebounds — is called catoptrics, and has so many uses, both marvelous and profitable, that it would take me too long to note the principal conclusions already known. And also, perhaps, some things might lack due credit with you, and I would lose my labor, and you would slip into a hasty judgment,

Before you have learned enough of the power of nature and art.

Astronomy



Now, to proceed: Astronomy is a mathematical art that demonstrates the distances, magnitudes, and all natural motions, appearances, and conditions proper to the Planets and fixed Stars: for any time past, present, and to come, with respect to a given Horizon, or without respect to any Horizon. By this art we learn the distance of the Starry Sky, and of each Planet from the Center of the Earth, and the size of any Fixed star seen, or Planet, in respect to the Earth's size. For example, we are sure, by this art, that the Solidity, Mass, and Body of the Sun contains the quantity of the whole Earth and Sea a hundred and thirty-two times, less by $\frac{1}{8}$, one eighth part of the earth. But the body of the whole earthly globe and Sea is bigger than the body of the Moon forty-three times, less by $\frac{1}{8}$ of the Moon. Therefore the Sun is bigger than the Moon 7000 times, less by $59 \frac{39}{64}$, that is, precisely $6940 \frac{25}{64}$ bigger than the Moon. And yet an unskilled person would judge them the same size. Therefore one of them must be much farther from us than the other. The Sun, when he is farthest from the earth (which now, in our age, is when he is in the 8th degree of Cancer), is 1179 semidiameters of the Earth distant. And the Moon, when she is farthest from the earth, is 68 semidiameters of the earth and $\frac{1}{3}$. The nearest the Moon comes to the earth is $52\frac{1}{4}$ semidiameters. The distance of the Starry Sky from us, in semidiameters of the earth, is $20081\frac{1}{2}$ — twenty thousand

eighty-one and a half. Subtract from this the Moon's nearest distance from the Earth, and there remain $20029\frac{1}{4}$ semidiameters of the earth — twenty thousand twenty-nine and a quarter.

So thick is the heavenly Palace that the Planets have all their exercise in, and most marvelously perform the Command and Charge given to them by the omnipotent Majesty of the King of kings. This is what in Genesis is called Ha Rakia. Consider it well. The semidiameter of the earth contains $3436\frac{4}{11}$ of our common miles, three thousand four hundred thirty-six and four elevenths of one mile, and so the whole earth and sea round about is 21600 of our miles, twenty-one thousand six hundred, allowing sixty miles for every degree of the greatest circle. Now if you weigh well with yourself just this little portion of the fruit of astronomy, concerning the size and distances of Sun, Moon, and Starry Sky, and the huge mass of Ha Rakia, will you not find your conscience moved, with the kingly Prophet, to sing the confession of God's Glory, and say, "The heavens declare the glory of God, and the firmament [Ha Rakia] shows the works of his hands." And so on, for those first five stanzas of that kingly Psalm. Well, well, it is time for some people to lay hold of wisdom, and to judge things truly, and not to explain the Holy word all by allegories, so as to neglect the wisdom, power, and goodness of God that is to be seen and learned in and by his Creatures and Creation. Through parables and analogies of their natures and properties, the course of Holy Scripture also declares very many Mysteries to us. The whole frame of God's creatures, which is the whole world, is to us a bright mirror: from which, by reflection, beams and radiations bounce back to our knowledge and perception, representing the image of his infinite goodness, almighty power, and wisdom. And we are by that taught and persuaded to glorify our Creator as God, and to be thankful for it. If the Heathens could find these uses in these most pure, beautiful, and mighty bodily creatures, shall we — after the true Sun of righteousness has risen above the Horizon of our temporal Hemisphere, and has so abundantly streamed the direct beams of his goodness, mercy, and grace into our hearts, a heat that all creatures feel, spiritual and bodily, visible and invisible — shall we, I say, look upon the Heaven, Stars, and Planets as an ox and a donkey do, caring and inquiring no further

about what they are, why they were created, and how they carry out what they were created for? Since all creatures were created for our sake, and both we and they were created chiefly to glorify the Almighty Creator, and to do that by every means possible to us. Do not be ignorant (says Plato in *Epinomis*) that astronomy is a thing of excellent wisdom. Astronomy was commended to us from the beginning, and in a manner commanded by God himself, since he made the Sun, Moon, and Stars to be for us Signs, and knowledge of Seasons, and for Distinctions of Days and years. Many words are not needed. But I wish everyone would weigh this word: Signs. And besides that, compare it also with the tenth chapter of *Jeremiah*. And though some think that they have found a rod there, yet Modest Reason will be an impartial judge of who ought to be beaten with it, as far as our purpose is concerned. Leaving that, please understand this: without great diligence of observation, examination, and calculation, their periods and courses — by which the Distinction of Seasons, years, and New Moons could be precisely known — could not be exactly certified. To do that is the art we have here defined to be astronomy. By it we may have the distinct Course of Times, days, years, and Ages: both for considering Sacred Prophecies foretold and accomplished in due time, and for holding high Mystical Solemnities, and for all other human affairs, conditions, and covenants at a set time between one person and another: with many other great uses: in all of which, truly, there would be great uncertainty, confusion, untruth, and brutish barbarism without the wonderful diligence and skill of this art: continually learning and fixing Times and periods of Time by the record of the heavenly book in which all times are written, and to be read with an astronomical staff instead of a pointer stick.

Music



Music, as a kind of motion, has its original cause. So, after the swiftest and slowest motions in the firmament, which nature performs and the astronomer considers, I will now speak of another kind of motion: sound-producing, audible, and countable by human beings. By music here I mean the science that the Greeks call Harmonice. I will not take up the dispute between the ancient harmonists and canonists. Music is a mathematical science. By sense and reason it teaches us to judge and order the differences between high and low sounds perfectly. Astronomy and music are sisters, Plato says. The eyes were made for astronomy, and the ears for harmonious motion. But astronomy has a more divine contemplation and usefulness than any mortal eye can perceive. So music is to be considered:

(1.)

that the mind may be preferred to the ear. And from audible sound we should go on to examine which numbers are in harmony and which are not. And why the one are and the other are not. I could at length,

(2.)

in the heavenly motions and distances, describe the marvelous harmony of Pythagoras' harp

(3.)

with eight strings. Something could also be said of Mercury's two harps,

(4.)

each with four elemental strings. And very strange things could be said about the harmony,

(5.)

that belongs to our spiritual part. As can be seen in Ptolemy's third book, in the fourth and sixth chapters.

(6.)

And what causes the fitting bond and friendly fellowship between the intellectual and mental part of us and our gross and corruptible body, but a certain middle and harmonious spiritual nature, in which both share and which (in a way) results from both? In

(7.)

the tune of the human voice, and also

(8.)

the sound of instruments: of what could be said about harmony, no ordinary musician would readily believe.

(I. D. Read Aristotle's 8th book of Politics: the 5, 6, and 7 chapters. There you will have further occasion to think about music than is commonly thought.)

But of the various mixture (as I may call it), the coming together, the different comparison, and the application of these harmonies: as of three, four, five, or more, the effects have been marvelous; and the like could still be found and produced, if we take some proportional account of our time and our situation, compared with the state of the things then, in which and by which those wonderful effects were produced. Democritus and Theophrastus said that by music the griefs and diseases of mind and body could be cured or brought on. And we find in the record that Terpander, Arion, Ismenias, Orpheus, Amphion, David, Pythagoras, Empedocles, Asclepiades, and Timotheus, by harmonic consonance, have done and brought about things more than marvelous to hear of. I will say no more about them here. I am sure that ordinary music, as commonly used, is found by musicians and listeners to be so useful and pleasant that, if I were to say and argue only this much—that it ought to be used differently from the way it is—I would find more critics than I could find people who under-

stood what I meant. So in things that are evident and better known than I can express, and that are accepted and approved (as I wish some other things had the same luck), I will not extend my text any further, but will go on with my purpose.

Cosmography



Here I will give you a brief account of cosmography. Cosmography is the whole and complete description of both the heavenly and the elemental part of the world, and how they correspond to each other, with the comparison between them that is needed. This art requires astronomy, geography, hydrography, and music. So it is no small art, nor as simple as it is casually taken to be in common practice. It matches heaven and earth in one frame, and fits their corresponding parts together properly, so that the heavenly globe can be drawn, in practice, on the geographic and hydrographic globe. There we can consider the equinoctial circle, the ecliptic line, the colures, the poles, the stars in their true longitudes, latitudes, declinations, and altitudes, as well as the climes and parallels. And with an attached horizon and the rotation of the earthly globe (as the heaven is carried around by the *primum mobile*, the outermost moving sphere, in 24 equal hours), we can learn the risings and settings of the stars (as prescribed by Virgil in his *Georgics*; by Hesiod; by Hippocrates in his medical sphere, to Perdicca, king of the Macedonians; by Diocles, to King Antigonus; and by other famous philosophers): a thing necessary for working the earth properly, for navigation, and for changes in the human body, whether healthy, sick, wounded, or bruised. By the rotation, too, or the movement of the cosmographic globe, the rising and setting of the sun, the lengths of days and nights,

and the hours and times (both night and day) are known, along with very many other pleasant and necessary uses. Of these, some are known; better ones remain for those who know how to use them:

Note this.

who can make a wonderful bonfire from a spark of true fire, by applying the right material properly.

Astrology



Here I make astrology an art separate from astronomy: not by any new device, but by good reason and authority. For astrology is a mathematical art that reasonably demonstrates the workings and effects of the natural beams of light and the secret influence of the stars and planets, in every element and in every elemental body, at all times, in any given horizon. This art is furnished with many other great arts and experiences: with perfect perspective, astronomy, cosmography, natural philosophy of the 4 elements, the art of graduation, and some good understanding of music; and moreover with another great art that follows later, though I set this one before it here, for reasons that move me. You can see that there is enough material to make this rare and secret art from, and that it is hard enough to shape into a syllogistic conclusion. Yet the many and unceasing labors of the most ancient and wise philosophers, to attain this art, and to confirm it by examples of its effects, have left us enough proof and witness: and we can also see every day that the human body, and all other elemental bodies, are altered, disposed, ordered, pleased, and displeased by the influential working of the sun, the moon, and the other stars and planets. So says Aristotle, in the first of his Meteorological books, in the second chapter: this elemental world is of necessity almost next adjoining the heavenly motions, so that from there all its force may be governed. For that is to be taken as the first cause of all things, from which the beginning of motion is. And again, in the tenth chapter: We must therefore also take up the principles of these things,

and the causes of all things alike. The principle, then, as the mover, the chief and first of all, is that circle in which the sun's motion is plain, etc. And so on. His Meteorological books are full of arguments and effective demonstrations of the force, working, and power of the heavenly bodies in and upon the four elements, and upon other bodies made of them, whether perfectly or imperfectly composed. And in his second book, *On Generation and Corruption*, in the tenth chapter: the uppermost motion is not the cause of generation and corruption, but the motion of the zodiac is, because that motion is both continuous and caused by two movements. And in his second book, and the second chapter of his *Physics*: For man, says he, and the sun, are the cause of man's generation. Very many authorities can be brought for that conclusion, of 1000, 2000, even 3000 years of antiquity, from great philosophers, expert, wise, and godly men: a conclusion that we can discern and perceive daily and hourly by sense and reason. All beasts feel it and simply show it in their actions and feelings, outward and inward. All plants, herbs, trees, flowers, and fruits. And finally, the elements, and all things made of the elements, give testimony, as Aristotle said, that their whole dispositions, forces, and natural motions depend on the activity of the heavenly motions and influences. So, besides the specific order and form due to every seed, and besides the nature proper to the individual matrix of the thing produced, what the heavenly impression will be is what the perfect and careful astrologer has to conclude: not only (by apotelesms) τὸ ὅτι, but by natural and mathematical demonstration τὸ διότι. What sciences are needed for this, without exception, I have partly warned you of here; and in my *Propædeumes*, besides other matter disclosed there, I have set out the whole method mathematically: something that in our age has not been so carefully handled by anyone I ever saw or heard of. I was,

(Years 1548 and 1549, in Louvain.)

(for 21 years ago) by certain earnest disputations with the learned Gerardus Mercator, Antonius Gogaua, and others, so provoked to this; and by my constant and unbreakable zeal for the truth, in observations of heavenly influences down to the minute of time, then so diligent; and chiefly by the supernatural influence from the Star of Jacob, so directed: that any modest and sober stu-

dent who carefully and diligently seeks the truth will both find and confess the truth in my words here, and will also become a reasonable reformer of three sorts of people who go greatly wrong about these influential workings.

The one sort is light believers, the other light despisers, and the third light practitioners. The first and most common sort think that the heavens and stars are answerable to any of their doubts or desires:

(1.)

That is not so, and in fact they reach much too far. The second sort think that no influential force from the heavenly bodies has any sway in generation

(2.)

and corruption in this elemental world. And to the sun, the moon, and the stars—so many, so pure, so bright, so wonderfully big, so far away, so varied in their motions, so constant in their periods, and so on—they assign a slight, simple office or two, and so allow them, according to their capacities, as much influential force and power as the sign of the Sun, Moon, and Seven Stars hung up as signs in London to tell the houses apart, and other such crude helps in our worldly affairs. And they do not understand, or will not understand, the other workings and forces of the heavenly sun, moon, and stars: not as much as the sailor or the farmer; no, not as much as the elephant, or the Cynocephalus, or the porcupine. Nor will they allow these perfect and uncorrupted mighty bodies as much forceful radiation and strength as they see in a small piece of magnet stone, which shows its working at a great distance. And perhaps they think that the sea and rivers, like the Thames, are some living thing, and so ebb and flow, and run in and out, of themselves, at their own fantasies. God help, God help. Surely these men fall short, and are either too dull, or willfully blind, or perhaps too malicious. The third sort is the common and ordinary astrologer or practitioner, who, though not duly, skillfully, and perfectly

(3.)

furnished, yet goes wrong, either for vain glory or gain, or like a simple fool and blind Bayard, in both matter and manner: to the discredit of the wary and modest astrologer, and to the robbing of those most noble bodily creatures of their natural force: they being most mighty, most beneficial to all elemental

generation, corruption, and their appurtenances, and most harmonious in their monarchy. For these things, known and used with due restraint, we might highly and continually glorify God, with the princely prophet, saying: “The heavens declare the glory of God: who made the heavens in his wisdom: who made the sun to have dominion over the day: the moon and stars to have dominion over the night: so that day to day utters speech, and night to night declares knowledge. Praise him, all you stars, and light. Amen.”

Statics, 1



In order, now follows, of Statics: some words on what we mean by that name, and what benefit depends on such an art. Statics is a mathematical art. It shows the causes of heaviness and lightness in all things, and the motions and properties that belong to heaviness and lightness. And because the balance, or scales, is the chief instrument of the senses, and experience of these demonstrations can be had through it, we call this art statics: that is, experiments with the balance. If only people knew what profit, in every way, this art could bring to a capable examiner and a diligent practitioner.

“You alone know all things exactly (O God), who have made weight and balance your judgment, who have created all things in number, weight, and measure, and have weighed the mountains and hills in a balance, who have weighed both heaven and earth in your hand. We are therefore warned by your sacred word to consider your creatures, and by that consideration to gain a glimpse, as it were, or a shadow of understanding, that your wisdom, power, and goodness are infinite and unspeakable, declared in your creatures. And being further warned by your merciful goodness that three principal means were used in the creation of all your creatures, namely number, weight, and measure; and since the two arts of number and measure, ancient, famous, and most necessary for human uses, are already sufficiently known and available; we ask you, through your accustomed goodness, that this third key may come to the full and sufficient knowledge of such servants of yours as would gladly find

in your workmanship your true designs (made purposely for that use), by which we should glorify your name and show your wondrous wisdom and goodness to those who are weak in faith. Amen.”

Do not be surprised at this outburst, dear friend, you gentle and zealous student. Another day you may see what moved me to it. For now I will give you some basis, and also some show, of the benefits that come from this art. And because this art is rare, my words and practices might be too obscure, unless some light were held before the matter. That will best be done by giving you a few main conclusions out of Archimedes’ demonstrations, as follows.

1 The surface of any liquid, standing by itself and at rest, is spherical. Its center is the same as the center of the Earth.

2 If solid bodies of the same size, or volume, as any liquid, and with the same weight, are lowered into that liquid, they will settle downward so that no part of them lies above the surface of the liquid, and yet they will not sink all the way down, or drown.

3 If any solid body lighter than a liquid is lowered into that liquid, it will settle down into it so far that the amount of liquid equal in volume to the part of the body settled into it weighs as much as the whole solid body.

4 Any solid body lighter than a liquid, pushed down into that liquid, will move upward with a force equal to how much heavier the liquid of the same volume as the whole body is than that body itself.

5 Any solid body heavier than a liquid, lowered into that liquid, will sink all the way down. In that liquid it will be lighter by exactly the weight of the liquid that has the same size, or volume, as the solid body.

6 (John Dee. A sphere can be cut to any assigned proportion by this proposition, mechanically, by mixing a liquid to a certain weight compared with the weight of the sphere floating in it.)

If any solid body lighter than a liquid is lowered into that liquid, the weight of that body is to the weight of the liquid (which is equal in volume to the whole body) in the same proportion as the part of the body that has settled down is to the whole body.

From these truths, great errors can be set right in people's opinions about the natural motion of light and heavy things. Almost everyone in natural philosophy allows these errors, trusting too much to authority and to false assumptions. For example, that of two bodies, the heavier moves downward faster than the lighter.

(A common error, noted.)

This error was not first pointed out by me, but by a man named John Baptist de Benedictis. His chief proposition is this, and it seems like a paradox:

If there are two bodies of one shape and one kind, equal in volume or unequal,

(A paradox.)

they move through equal spaces in equal times, whether both are moving in air, or both in water, or in any one medium.

Hereupon, in the art of gunnery,

certain good discussions, otherwise, may receive great improvement and advancement.

(The wonderful uses of these propositions.)

Also, in the intended purpose, allow something for nature's imperfection, which does not match the precision of a demonstration. Moreover, by the propositions above, wisely used, air, water, earth, and fire can be measured closely: how light or heavy they are naturally, in their assigned parts, or as a whole. Then, turning your practice to elemental things, you can work out the proportions of the elements in compound things. Then the proportions of the humors in a person: their weights, and the weight of their bones and flesh, and so on. Then, by weight, you can consider a person's strength in any way, whole or in part. Then you can consider, with pleasure, how differently ships draw water, in the sea and in fresh water, and how to weigh up anything sunk in the sea or in fresh water, and so on. And, to lift your head up high, by weight you can measure the diameters of the Sun and Moon as precisely as by any other instrument, and so on. Friend, please weigh these things with the fair balance of reason, and you will find wonders upon wonders. You will value one drop of truth, even in natural philosophy, more than whole libraries of opinions that

are undemonstrated, or that do not answer to nature's law and your experience. Leaving these things there, I will give you two or three simple practices that serve a great purpose, and so finish my notes on statics. In mathematical matters, with the help of the mechanic, we will see here the value of weight.

(A practical exercise in statics, to find the proportion between the cube and the sphere.)

Make a cube of one uniform, evenly heavy material. From that same material, make a sphere, or globe, with a diameter exactly equal to the edge of the cube.

Your material may be wood, copper, tin, lead, silver, and so on (being, as I said, of the same nature and condition, and of even weight throughout).

And with an assay balance you may have prepared a great number of the smallest weights, weights that balance can tell apart or test. In that way you can go on to make up a complete set, a company and number of weights, up to six, eight, or twelve pounds in weight, every one of them most carefully tested. And know the amount of each one in your smallest weighable weight.

[Those who cannot get weights of precision can make a number of weights fairly close to precision from uniform, well-sifted sand, by always halving the sand; in time they will come to a smallest common weight. On that I leave the further matter to the judgment of those whom need compels.]

The Venetian way of reckoning weight may seem precise enough: by eight steps in a descending series, halving from a single grain.

(John Dee. For so you have 256 parts of a grain.)

Your cube, sphere, suitable balance, and suitable weights being ready, get to work. First, weigh your cube. Note the number of weights. Then weigh your sphere, and note the number of weights again. If you now find the weight of the cube to be to the weight of the sphere as 21 is to 11, then you see how the mechanic and the experimenter, without geometry and demonstration, are taught in effect the proportion of the cube to the sphere, as I have demonstrated at the end of the twelfth book of Euclid. Try often with the same cube

and sphere. Then change the sphere and the cube to another material, or another size, until you have made a complete general test of it. You may well come closer to the true terms of the proportion.

When you have found this one certain drop of natural truth, go on to draw conclusions, and properly test the matters that depend on it. For it is well demonstrated that a cylinder whose height and the diameter of its base equal the diameter of the sphere is sesquialter to that sphere, that is, as 3 to 2. To the weight of the sphere, add half as much again, and you have the weight of that cylinder. That cylinder is also contained in our earlier cube, so that the base of that cylinder is a circle drawn inside the square that is the base of our cube. But the cube and the cylinder, being the same height, have their bases in the same proportion as they bear to each other in bulk, or solidity. But before that, we have two numbers expressing their bulk, solidity, and quantity by weight; therefore,

(The proportion of the square to the inscribed circle.)

we have the proportion of the square to the circle inscribed in that same square. And so we come to a sensible and practical knowledge of Archimedes' great secret, sought and found by him through great labor of mind. Therefore, to any given circle, you can give an equal square:

(Squaring the circle mechanically.)

as I have taught in my annotation on the first proposition of the twelfth book. Likewise, to any given square, you can give an equal circle:

(To any given square, to give an equal circle.)

If you draw a circle that stands in the same proportion to your inscribed circle as the square does to that same circle, you can do this by my annotations on the second proposition of the twelfth book of Euclid, in my third problem there. Your diligence may bring you to a proportion of the square to the inscribed circle nearer the truth than 14 to 11. And notice that you can begin with the circle and the square, and so work out what proportion the sphere and the cube bear, just as you have now come from the sphere to the circle. For, if you make a square figure from silver, or gold, or brass sheets or plates (drawn through one hole, as is the custom), and weigh it; then describe the inscribed

circle on it, and cut off and file away, precisely down to the circle, the surplus of the square; then, weighing your circle, you will see whether the weight of the square is to your circle as 14 to 11. As I have noted at the beginning of Euclid's twelfth book, and so on. After that, go to my last proposition, on the last of the twelfth, and help yourself through to the end. And note this in passing.

(Note: squaring the circle without knowing the proportion between the circumference and the diameter.)

That we can square the circle without knowing the proportion of the circumference to the diameter, as you have just seen. And I can demonstrate it in other ways too. So many people have burdened themselves needlessly by working first on that point, which did not need to come first, and is also very intricate. And you can easily, in several different ways, come to know the circumference, once the circle's size is known first. I leave that to your consideration, and hurry on to dispatch another masterly problem, and to bring it nearer to your knowledge, and easier for you to handle, than the world has had it before this day, so far as I can tell. And that is, a mechanical doubling of the cube, and so on. It can be done like this:

(To double the cube readily, by mechanical art, depending on mathematical demonstration.)

Make a square upright pyramid, or a cone, of copper plates or tin plates, perfectly shaped hollow on the inside. In this, take great care to approach, as near as possible, the mathematical perfection of those figures. Let their bases be open all around; everywhere else, as closed and exact as possible. From the vertex to the circumference of the base of the cone, and to the sides of the base of the pyramid:

(John Dee. The 4 sides of this pyramid must be 4 matching equal isosceles triangles.)

let 4 straight lines be drawn on the inside of the cone and the pyramid, so that where they fall on the perimeters of the bases they make equal angles on both sides of themselves with those perimeters. Divide these 4 lines (in the

pyramid, and as many in the cone): one into 12 equal parts, another into 24, another into 60, and another into 100, counting up from the vertex. Or use other numbers of divisions, as experience teaches you.

(John Dee. In all work with this pyramid or cone, keep their positions alike in every point and condition while you are doing one job. Otherwise you will go wrong.)

Statics, 2



Then set your cone or pyramid, with the vertex downward, perpendicular to the base. (It does not matter if it is the other way around; it hinders nothing.) So let them be kept as steady as possible. Now, if there is a cube that you want to double: make a small cube of copper, silver, lead, tin, wood, stone, or bone. Or else make a hollow cube, or a cubic box, of copper, silver, tin, or wood, etc. You may size these in respect of your pyramid or cone so that the pyramid or cone can hold 3 or 4 times their weight in water, at least, whatever material they are made of. Do not let your solid angle at the vertex be too sharp, so that the water can easily reach the very vertex of your hollow cone or pyramid. Put one of your solid cubes in a suitable balance and take its weight exactly in water. Pour that water (without loss) quietly into the hollow pyramid or cone. Mark in your lines what numbers the water cuts. Take the weight of the same cube again, in the same kind of water you had before:

John Dee. Consider well when you must put your waters together, and when you must empty your first water out of your pyramid or cone. Otherwise you will err.

Put that also into the pyramid or cone where you put the first. Mark again in what number or place of the lines the water cuts them. There are two ways to reach your aim: that is, either by numbers or by lines. By numbers: divide the side of your fundamental cube into as many equal parts as it can conveniently hold, with ease and precision in the division. For as the number of your first

and lesser line (in your hollow pyramid or cone) is to the second and greater one, both counted from the vertex, so the number of the side of your fundamental cube is to the number belonging to the radical side of the cube that is double your fundamental cube. Multiply that number cube-wise and it will soon show whether it is double the cube number of your fundamental cube or not. By lines, thus: as your lesser and first line (in your hollow pyramid or cone) is to the second and greater, so let the radical side of your fundamental cube be to a fourth proportional line, by the 12th proposition of the sixth book of Euclid. That fourth line will be the cube root, or the radical side, of the cube that is double your fundamental cube, which is what we wanted.

God be thanked for this invention and the fruit that follows.

For this I may say with joy, “I have found it, I have found it, I have found it,” thanking the holy and glorious Trinity, having greater cause for it than

Vitruvius, book 9, chapter 3.

Archimedes had for finding the fraud used in the king’s crown of gold, as anyone can easily judge by the difference of the results that follow from the one and the other. Where I spoke before of a hollow cubic box: the same use is made of it, and without weighing. Thus: fill it with water precisely full, and pour that water into your pyramid or cone. And here note where the lines are cut in your pyramid or cone. Again, fill your box as you did before. Put that water also to the first. Mark the second cutting of your lines. Now, as you proceeded before, so you must proceed here.

Note.

And if the cube you are to double is ever so great, you have in this way the proportion (on a small scale) between your two little cubes. And then the side of that great cube (to be doubled), being the third, will have a fourth found that is proportional to it, by the 12th of the sixth of Euclid.

Note concerning the spherical surface of the water.

Note that all this while I have not forgotten my first proposition in statics, restated here: that the surface of the water is spherical. In this use your judgment: to the first line add a small hairbreadth more, and to the second add half

a hairbreadth more to its length. For you will easily see that the difference can be no greater in any pyramid or cone you are to handle. And you can try it this way, for finding how much the water swells above the level.

“Square the semidiameter from the center of the earth to the surface of your first water. Then square half the subtendent of that watery surface (the subtendent must have its measure divided into equal parts, the same as the semidiameter from the earth to your watery surface). Subtract that square from the first. Of the remainder, take the square root. Subtract that root from your first semidiameter from the earth to your watery surface: what remains is the height of the water in the middle, above the level.” You will find this to be something too small to notice. And even if it were greatly noticeable,

Note.

Yet, by help of my sixth theorem on the last proposition of Euclid’s twelfth book, noted, you may reduce everything to a true level. But you must take further care against accidental causes of the water swelling: for example, if you dampen your hollow pyramid or cone beforehand a little with a moist sponge, you will prevent an accidental cause of swelling, etc. Experience will teach you in abundance, with great ease, pleasure, and benefit.

Thus you can double the cube mechanically, treble it, and so on, in any proportion.

Note this abridgment of doubling the cube, etc.

Now I will shorten your effort, cost, and care in this. Without any preparation of your fundamental cubes, you may work this conclusion in the same way. For that was rather a kind of experimental demonstration than the shortest way, and it all depended on one mathematical demonstration: “Take water (as much as will conveniently serve your purpose, as I warned earlier about the size of your fundamental cubes). Weigh it precisely. Put that water into your pyramid or cone. Then take again the same kind of water, the same weight you had before, and put that also into your pyramid or cone. For each time, your marking of the lines, how the water cuts them, will give you the proportion between the radical sides of any two cubes, one of which is double the other, working as I taught you before:

Note.

Except that, for your fundamental cube's radical side, you may take any straight line you like here."

Yet a step further with our drop of natural truth:

To give cubes one to another in any proportion, rational or irrational.

You may now give cubes one to another in any proportion given, rational or irrational, in this way. Make a hollow parallelepipedon of copper or tin, with one base missing, or open, as in our cubic box. From the bottom of that parallelepipedon raise up many perpendiculars on each of its four sides. Now if any proportion is assigned to you in straight lines, cut one of your perpendiculars (or a line equal to it, or shorter than it) likewise, by the 10th proposition of the sixth book of Euclid. Set those two parts on two separate lines of those perpendiculars (or you may set them both on one line), making their beginnings at the base, so that their lengths extend upward.

Now set your hollow parallelepipedon upright, perpendicularly, and steady. Pour in water neatly to the height of your shorter line. Pour that water into the hollow pyramid or cone. Mark the place it rises to. Set your hollow parallelepipedon again. Pour water into it exactly to the height of the second line.

Empty out the first.

Pour that water properly into the hollow pyramid or cone. Mark now again where the water cuts the same line you marked before. For there, as the first marked line is to the second, so the two radical sides will be to each other, of any two cubes that in their solidity have the same proportion that was first assigned, whether rational or irrational.

Thus, in various ways, you may equip yourself with such strange and profitable matter, which has long been wished for. And though it is done naturally and mechanically, it still has a good mathematical demonstration.

The demonstrations of this doubling of the cube, and of the rest.

Which is this: you always have two similar pyramids, or two similar cones, in the proportions assigned; and similar pyramids or cones stand to one another in the proportion of their homologous sides (or lines) tripled. Therefore, if to the first and second lines found in your hollow pyramid or cone

you join a third and a fourth in continued proportion, that fourth line will be to the first as the greater pyramid or cone is to the lesser, by the 33rd of the eleventh of Euclid. If pyramid to pyramid, or cone to cone, is double,

John Dee. By this, help yourself become a precise practitioner. And so consider how the convexity of the water does not hinder you at all, in any way you can notice.

Then line to line will also be double, etc. But as our first line is to the second, so is the radical side of our fundamental cube to the radical side of the cube to be made or to be doubled. And therefore, joining a third and a fourth line in continued proportion to those two as well will give a fourth line in that proportion to the first as our fourth pyramidal or conic line was to its first. But that was double, or treble, etc., as the pyramids or cones were to one another (as we have proved). Therefore this fourth will also be double or treble to the first, as the pyramids or cones were to one another. But the cube we make is generated from the second of the four proportional lines:

By the 33rd of the eleventh book of Euclid.

Therefore, as the fourth line is to the first, so that cube is to the first cube. And we have proved the fourth line to be to the first as the pyramid or cone is to the pyramid or cone. Therefore the cube is to the cube as pyramid is to pyramid, or cone is to cone.

John Dee. And your diligence in practice can perform it in this way (by the weight of water). Therefore you are now able to give a good reason for all you do.

But we suppose pyramid to pyramid, or cone to cone, to be double or treble, etc. Therefore cube is to cube double, or treble, etc. Which was to be demonstrated. And for the parallelipedon it is evident that the solid parallelipedons of water stand to one another as their heights do, since they have one base. Therefore the pyramids or cones made from those water parallelipedons stand to one another as the lines stand to one another between which our proportion was assigned. But the cubes made from lines, in the proportion of the pyramidal or conic homologous lines, stand to one another as the pyra-

mids or cones stand to one another (as we proved before). Therefore the cubes made will stand to one another as the assigned lines stand to one another. Which was to be demonstrated. Note.

Note this corollary.

This demonstration of mine is more general than only in a square pyramid or cone: consider it well. Thus I have been very long-winded, both mathematically and mechanically; yet, I trust, nothing tedious to those who are well disposed toward these things. And in truth I am forced (to avoid prolixity) to omit several such things, easy to practice: they would be a great treasure to the mathematician, and no small gain to the mechanic.

The great benefits that follow from these new inventions.

Now, between two given lines, you may find two mean proportionals in continued proportion, by the hollow parallelipedon and the hollow pyramid or cone. Now, any rectangular parallelipedon being given, three straight lines may be found, proportional in any proportion assigned, out of which a parallelipedon equal to the given parallelipedon will be produced. Of this I noted something on the 36th proposition of the 11th book of Euclid. Now, all those things that Vitruvius in his Architecture specifies as able to be done by doubling the cube, or by finding two mean proportional lines between two given lines, can easily be performed. Now, that problem which I noted to you at the end of my addition on the 34th of the 11th book of Euclid is proved possible. Now, any regular body may be transformed into another, etc. Now, any regular body, any sphere, yes, any mixed solid, and what is more, irregular solids, may be made (in any proportion assigned) like the body first given. Thus, from a little figure (as the Dutch painters call it) in the same symmetry, a giant can be made, and that with any gesture used by the little figure, and the other way around. Now, from any mold or model of a ship you may make one, of the same mold (in any assigned proportion), bigger or smaller.

Now, you may, of any gun, or small piece of ordnance, make another with the same symmetry (in all points) as big and as small as you want. Mark that and think on it. You may apply this infinitely, something so long sought for and

now so easily worked out, and at the same time so willingly and freely shared with those who deal faithfully with virtuous studies.

Such is the fruit of the mathematical sciences and arts.

Statics, 3



Thus the mathematical mind can deal speculatively in its own art, and by good means climb above the clouds and stars. And thirdly, it can, in order, descend, to shape natural things for wonderful uses, and when it wishes return home to its own center, and there prepare more means to ascend or descend by, all to the glory of God and our honest delight on earth.

Although the printer has waited for this preface a day or two, I could not take my pen from the paper before I had given you encouraging word and brief instructions on some of the benefits that can be reaped by statics. In the rest I will therefore be as brief as possible, and at the same time describe them somewhat to match. And you will see this in what comes next in order. For although it is so ample and wonderful that one could find fruitful matter in it to speak of for a whole year, and in practice is a treasure without end, yet I will glance over it with very few words.

Anthropography



This I call Anthropography. Which is an art restored, and which I put forward for your service. Please think of it as one of the chief points of human knowledge. Although it has only now first been confirmed with this new name, the matter has been considered from the beginning by all perfect philosophers. Anthropography is the description of the number, measure, weight, figure, situation, and color of every diverse thing contained in the perfect body of man: with certain knowledge of the symmetry, figure, weight, characterization, and proper local motion of any assigned part of the said body, and of the numbers belonging to that part. This is the part of the definition that fits this place: enough to show the particularity and excellence of the art, and why it is ascribed here to the mathematical sciences. If the description of the heavenly part of the world has its own art called astronomy: if the description of the earthly globe has its own art called geography: if the matching of both has its own art called cosmography, which is the description of the whole and universal frame of the world: why should not the description of

Man is the lesser world.

him who is the lesser world, and from the beginning called Microcosmus (that is, the lesser world), and for whose sake and service all other bodily creatures were created, who also shares with spirits and angels, and is made in the image and likeness of God, have its own art, and be called the art of arts, rather than either lack a name or have too base and improper a name? You must bor-

row or claim for yourself particular parts of this from various professions, and proceed further, as God, nature, reason, and experience shall instruct you. The anatomists will restore some part to you: the physiognomists some: the chiromantists some. The metaposcopists some: the excellent Albert Durer a good part: the art of perspective will help forward the eye somewhat: Pythagoras, Hippocrates, Plato, Galen, Meletius, and many others (in certain things) will contribute. And further, the heavens, the earth, and all other creatures will each show and offer their harmonious service to fill up whatever is lacking here: and concluding with your own experience, you may methodically register the whole for posterity: by which good proof will be had of our harmonious and

Microcosmus.

Microcosmic constitution.

The outward image and view of this, for the art of drawing and painting, for sculpture, and for architecture (for a church, house, fort, or ship), is most necessary and profitable, since it is the chief base and foundation of them.

Book 3, chapter 1.

Look in Vitruvius and see whether I am dealing sincerely for your benefit. Look in Albert Durer, *On the Symmetry of the Human Body*. Look in the 27th and 28th chapters of the second book, *On Occult Philosophy*. Consider the ark of Noah. And from that, wade further. Remember the Delphic oracle “know yourself”, pronounced so long ago, repeated by so many a philosopher, and attempted by the wisest: and then you will see how long ago you have been called to the school where this art could be learned. Well, I am not at all afraid of the disdain of some who think the sciences and arts are only seven. Perhaps such people may, with enough ignorance and shame, fall short of those seven as well: and yet, nevertheless, they cannot set a certain number of arts, nor set in each certain impassable bounds against God, nature, and human industry. New arts rise up daily, and there was no order taken that,

All arts should be made known to the world in one age, or in one land, or by one man.

Let us embrace the gifts of God, and the ways to wisdom, continually bestowed from above in this time of grace on those who will receive them thankfully: and to the good, all things will work together for good.

Trochilike



Trochilike is the mathematical art that shows the properties of all circular motions, simple and compound. Because its fruit, as people commonly receive it, is in wheels, it takes the name Trochilike: what you might call wheel art. With this art you can set a wheel to make one full turn in any given time. You can also set two wheels so that their turns in the same time, or in equal times, stand in any proportion you choose. With wheels you can describe a straight line. You can also draw a spiral line on a flat plane, conic section lines, and other irregular lines, as you please. These and similar results are the main conclusions of this art, and they bring forward many pleasant and profitable mechanical works:

(Saw mills.)

Such as mills that saw great and very long deal boards, with no one standing by. I have seen such in Germany, and in the city of Prague, in the kingdom of Bohemia: coining mills, hand mills for grinding grain, and all manner of mills and wheelwork, driven by wind, smoke, water, weight, springs, people, or animals. Take Agricola's *On the Nature of Metals* in hand and you will see, in every mine, how great the need for wheelwork is. With wheels, strange and incredible works are done, as will appear in other arts later. A wonderful example of still greater possibility, and of present use, appeared in my time in a certain instrument. Its inventor and craftsman had sold it earlier for twenty talents of gold, and it had then suffered injury and damage by misfortune. Ianellus of

Cremona repaired it and presented it to Emperor Charles V. Hieronymus Cardanus can be my witness that in it there was one wheel which turned at such a rate that in 7000 years only it would complete its own period. That is almost incredible. How much further it goes, I keep within my bounds: very many men, still alive, can tell.

Helicosophy



Helicosophy is a near sister to Trochilike. It is the mathematical art that shows how to design all spiral lines on a flat plane, on a cylinder, a cone, a sphere, a conoid, and a spheroid, with the properties that belong to them. Its use in architecture and in various instruments and engines is essential. In many things the screw does the work that could not otherwise be done. With its help,

(Atheneus, Book 5, chapter 8.)

It is recorded that, when all the power of the city of Syracuse could not move a certain ship that lay aground, the mighty Archimedes set his screw engine to work and made King Hiero move it himself, at ease, as he wished.

(Proclus, page 18.)

The king was astonished at this: *Απὸ ταύτης τῆς ἡμέρας, περὶ παντός, Αρχιμήδει λέγοντι πιζευτεόν.* “From this day forward,” said the king, “Archimedes ought to be believed in whatever he says.”

Pneumatithmie



Pneumatithmie shows, by closed hollow geometric figures, regular and irregular, the strange properties of water, air, smoke, and fire, in motion or at rest, within their own continuity and as they are joined to the elements next to them. This art is very useful to the natural philosopher, because it proves that a vacuum, or empty space, does not exist in the world. It also shows that all nature dislikes it so much that, contrary to ordinary law, the elements will move or stand still. Water rises rather than leave a space between itself and the air, more than that quantity of air naturally requires or can fill. Again, water hangs and does not fall rather than fall and leave emptiness behind it. Fire and air act the same way: they will descend rather than let their continuity be broken or the element next to them be forced away. And just as they will not stretch into discontinuity, so they will not, and no human force can, be pressed or cramped into a space too small and unfit for their bodily substance. They will use great force and violence to keep their natural right and freedom.

(Going to the bottom of the sea without danger.)

On this basis, two or three men together can keep air under a great cauldron, press it down in order, and descend to the bottom of the sea without harm, staying there for a time, and so on. Note how the thicker element, water, gives way to the thinner, the air, and in turn takes force from the thinner, and so on. Pumps and all kinds of bellows are founded on this art, along with many other strange devices, such as hydraulica, organs driven by water, and so on.

Fine works on this subject, commonly called pneumatics, exist in both Greek and Latin. Old and learned scholars call it the Science of the Full and the Empty.

Menadrie



Menadrie is a mathematical art. It shows how power and force can be multiplied beyond nature's simple power, so that you can direct, lift, pull toward you, and push or throw any power, weight, or force, simple or multiplied and fixed in amount, that nature would not otherwise let you direct or move.

Other arts help this art a great deal: perspective in some points, statics in others, Trochilike in others, and Helicosophy and Pneumatithmie in others.

By this art, every crane, gibbet, and engine for lifting something up, or for forcing it in any way, is managed, and the exact cause of its force is known. One example is the force one man has with a Dutch wagon rack, enough to set upright again a heavy loaded wagon that has been overturned.

The force of the crossbow rack is clearly demonstrated here.

It also gives the reason why one man can lift with a lever what six men, with only their hands, could not do as easily.

By this art, in the ordinary cranes in London, where the power is to crane up a weight of 2000 pounds, adding two more wheels in good order, the art concludes that a weight of 200000 pounds can be craned up, etc.

Archimedes knew this art so well that he alone, with his devices and engines, twice or three times routed and defeated the whole army and host of the Romans besieging Syracuse,

(Plutarchus in Marco Marcello.)

Marcus Marcellus the Consul was their general captain.

(Synesius, in his Letters.)

With his engines he hurled such huge stones out of the city and among them: so many, with such force, and from so far away.

(Polybius.)

(Plinius.)

(Quintilianus.)

(T. Liuius.)

And at sea likewise: though their ships could come right up to the walls of Syracuse, he completely routed the Roman fleet. Partly by hurling huge stones:

(Athenæus.)

Partly by casting pikes 18 feet long, made like shafts, which he drove almost a quarter of a mile. Partly by catching hold of their ships, hoisting them up above the water, and suddenly letting them drop back into the sea:

(Galenus.)

(Anthemius.)

Partly by his burning glasses, with which he set their other ships on fire at a distance. And partly by his other schemes, devices, and engines. He conducted himself so bravely that all the force, courage, and strategy of the Romans, for a long time, could accomplish nothing toward taking Syracuse. So the Romans called Archimedes Briareus and Centimanus. Zonaras mentions a certain Proclus, who had understood the art of Menadrie so well, and had invented so well on his own, that with his burning glasses,

(Burning Glasses.)

placed on the walls of Byzantium, he multiplied the heat of the sun so, and directed its beams against his enemy's fleet with such force and so suddenly, like lightning, that he burned and destroyed both man and ship. Dion also specifies Priscus, a geometrician in Byzantium, who invented and used sundry engines of force multiplied. This was the reason the Emperor Severus spared his life after he had taken Byzantium, because he honored the art, wit, and rare industry of Priscus. But the invention of guns was in no way inferior to the invention of these engines of force.

(Gunnes.)

Guns got the occasion and order of their first invention from an Englishman, though they were first carried out in another land and by other men. And whoever saw the record where the general cause and order of gunnery is first discussed would think that “small things, slight and common, coming to wise men’s consideration and industrious men’s handling, may grow to be of force incredible.”

Hypogeiodie



Hypogeiodie is a mathematical art. It shows how, under the spherical surface of the earth, at any depth, a set course can be laid out and traveled toward any given perpendicular line (whose distance from the perpendicular at the entrance, and its azimuth likewise with respect to that entrance, are known). It shows how any course marked out above the surface of the earth can be kept underground at any set depth, always running directly beneath the course marked on the ground above. And the other way round: given any course under the earth, straight or crooked, how to lay it out on the outer face or surface of the earth, so that perpendiculars drawn from the center of the earth to the spherical surface land exactly on the matching points of those two courses. This art demonstrates all of that, along with every other case and circumstance involved, and everything that belongs to them. The art is very rich in the variety of its conclusions, and it is useful in many ways to the commonwealth. I invented this art at the request of two gentlemen. They had a certain working underground (for gain), and their plots of ground joined above the work; and because of the twists, the different depths, and the heights of the way underground, they were in doubt, and at odds over whose ground the working then lay. Before this, I had published only the name: *On the Subterranean Journey*. The rest is at God's will. For laborers, miners, and diggers for metals, stone, and

coal, and for secret passages underground from place to place (as this land has several), and for other purposes, anyone can easily see both the great fruit of this art and the great help geometry gives to it.

Hydragogie



Hydragogie shows that water can be led, by nature's law and with artificial help, from any source (a spring, standing water, or running water) to any other assigned place. This art has long been in use, much has been written about it, and very marvelous works have been accomplished in it, as can still be seen in Italy by the ruins of the aqueducts that remain. In other places there are rivers led through the mainland, navigable for many miles. In other places there are the marvelous forcings of water to rise. All of this shows the great skill required of anyone who would be perfect in this art, for every possible case of leading water. I need not speak of allowing fall for every hundred feet, or of vent holes (if the water's journey is long and hard), since many experienced men close at hand (around us) can testify well enough to the practice, even though they do not know the demonstration of why it is necessary. Nor, if they were led up and down and around the mountains from the source of the spring, and then, a place being given, they were asked how low or high that last place is compared with the source from which they had come by such a crooked, up-and-down route, perhaps they would not, or could not, answer that question readily or exactly. So geometry is necessary to Hydragogie. The various ways of forcing water to rise, by tympanum, kettle mills, screw, Ctesibian devices, or the like, can be seen in full in Vitruvius, Agricola, and others. And so it is also perfectly

clear how the arts of Pneumatithmie, Helicosophy, statics, Trochilike, and Menadrie supply this art in theory, and serve the good of the commonwealth in practice.

Horometry



Horometrie is a mathematical art. It shows how, at any appointed time, the exact customary reckoning of time can be known for any given place. These words are smooth, plain, easy English, but their meaning reaches further than you would readily imagine. Some part of this art was called Gnomonice in old times, and more recently Horologiographia, and in English it can be called dialing. The use is ancient, and the invention is older still. The use seems to have been in place at least more than two thousand three hundred years ago:

(4. Reg. 20.)

in King Achaz's dial, then, showing by the sun the divisions of time. This dialing can be done by sun, moon, and stars, and the exact time of day or night known. But drawing these dials demonstratively, of all kinds, requires good skill both in astronomy and in elemental, spherical, phenomenal, and conic geometry. Then, to use the foundations of the art on any regular surface that comes to hand in any place, and on it (in any possible fitting position) to set out every way in which the usual hours can be truly determined by the sun's shadow, will be found to be no easy painter's work. That means painting and setting out the sun's motion to the width of a hair. In this feat (in my youth) I invented a way: in any horizontal, mural, or equinoctial dial, etc., at all hours (when the sun shines), the sign and degree ascending can be known. This is very necessary for the rising of those fixed stars whose influence in the air is plainly very great. I will say no more about its use. But since people's affairs require knowing times

and moments when neither sun, moon, nor star can be seen, mechanical skill first invented how the time and hours could be known by water running at a steady rate. The famous Ctesibius was the inventor of this, a man whom Vitruvius rightly praised to the skies. After that, hours were measured by running sand. Then by Trochilike with a weight, and more recently by Trochilike with a spring, without a weight. All of these need oversight and correction by the direction of the sun or the stars (at set times), according to the heavenly equinoctial motion, besides the unevenness of their own working. There remains among the philosophers (not speaking figuratively here),

(A perpetual motion.)

a way more excellent, more useful, and more marvelous than all these: to have the motion of the Primouant (or the first equinoctial motion) imitated by nature and art. You will understand more of this later, through further search in weightier studies. And so it is time to finish this discussion of the divisions of time used in our public and private affairs. No man who knows how to spend his time would want to be without its benefit.

Zography



Drawing is a mathematical art. It teaches and shows how the intersection of all the visual pyramids, made by any assigned plane (with the center, distance, and lights determined), can be represented by lines and the proper colors. This is a remarkable art, and it would take a whole volume to set out its nature and the benefits that follow from it. For perfection, the draftsman needs great skill in geometry, arithmetic, perspective, and anthropography, along with many other particular arts. For the most excellent painter (who is only the draftsman's own craftsman and imitator of what the senses show) has reached such perfection that the senses of men and beasts have judged painted things to be natural rather than made by art: alive and not dead. This mechanical draftsman (commonly called the painter) is marvelous in his skill, and seems to have a certain divine power: he can make absent friends a warm, present comfort; yes, and he can give dead friends a lasting, silent presence, not only with us but with our descendants for many ages. And going on: consider how, in winter, he can show you a lively view of summer's joy and riches, and in summer present the face of winter's dreary state and bareness. Cities, towns, forts, woods, armies, yes whole kingdoms, however far away or great, he can easily bring home with him (into anyone's judgment) as living patterns of the things he has listed. In one little house he can enclose (to the great pleasure of the viewers) a living picture of all visible creatures: those living on the earth or under it; those in the waters lying, creeping, sliding, or swimming; and any bird or fly flying in the

air. Even set beside the stars, the sky, and the clouds, yes in the very look of light itself (that divine creature), he can match our eyes' judgment very closely. What a thing this is! Things that do not yet exist he can represent so well that, when they do exist, the picture seems (in a way) to have created them. To what craftsman is painting not a great pleasure and benefit? Which of them all would refuse the guidance and help of painting? The architect, the goldsmith, and the tapestry weaver all think a great deal of painting. Our vivid herbals, our portraits of birds, beasts, and fish, and our detailed anatomical drawings: how are they most perfectly made, or looked at with most pleasure? Is it not by painting alone? And if painting, through the painter's skill, is so useful and marvelous, what should be thought of drawing, the schoolmaster of painting and its chief governor? Though I do not mention sculpture in my table of mathematical arts, everyone can see how painting and sculpture are full sisters, and both very useful in a commonwealth. And excellent craftsmen have written large books in praise of sculpture as well as painting. I take as my witnesses Georgio Vasari, Pittore Aretino, Pomponius Gauricus, and others. To these two arts (and others) a certain odd art called Althamasat owes much: more than the ordinary sculptor, carver, engraver, cutter, graver, founder, or painter, etc., know how useful their own art is.

Architecture



(An objection.)

Many people may think that architecture is not worthy, or not fit, to be counted among the mathematical arts. To them I think it good to give some account of my doing so. Not worthy, they will say, because it is only for the building of a house, palace, church, fort, or such coarse works. And you also defined the mathematical arts to be such as dealt with no material or corruptible thing, and also proceeded demonstratively in their faculty, by number or magnitude. First,

The answer.

you see that I count architecture here among those mathematical arts which are derived from the principals; and you know that such may deal with natural things and sensible matter. Of which, “some come nearer to the simple and absolute mathematical speculation than others do.”



And though the architect plans, informs, and directs the craftsman in handiwork, and in the actual building of a house, castle, or palace, and is chief judge of the same, yet with himself (as chief master and architect) remains the demonstrative reason and cause of the craftsman’s work: in line, plane, and solid; by geometric, arithmetic, optic, musical, astronomical, cosmographic means (and to be brief) by all the earlier derived mathematical arts, and other natural arts, able to be confirmed and established.

If this is so, then you may think that architecture has good and proper standing in this honorable company of derived mathematical arts. I will here ask the judgment of two most perfect architects: one is Vitruvius, the Roman, who wrote ten books of it to the Emperor Augustus (in whose days our heavenly Archemaster was born); and the other is Leo Baptista Albertus, a Florentine, who also published ten books of it.

Architecture, says Vitruvius, is a science adorned with many doctrines and diverse teachings: by whose judgment all works, finished by other craftsmen, are judged.

It follows: Architecture grows out of framing and reasoning, etc. Reasoning is that which can demonstrate and plainly declare things framed with skill and a sense of proportion.

Again: In all things, and therefore chiefly in architecture, there are these two: the thing signified, and the thing that signifies. The thing proposed, of which we speak, is the thing signified. But demonstration, set out with the reasons of diverse doctrines, signifies the same thing.

After that: An architect, he says, should understand languages, be skilled in painting, well instructed in geometry, not ignorant of perspective, equipped with arithmetic, know many histories, have diligently listened to philosophers, have skill in music, not be ignorant of medicine, know the answers of lawyers, and have good knowledge of astronomy and the courses of the heavens.

He gives the reasons, in order, why all these arts, doctrines, and instructions are required in an excellent architect. And, for brevity, leaving out the Latin text, he has this:

Secondly, an architect needs the knowledge of painting, so that he can more easily shape out, in painted patterns, the form of whatever work he likes. And geometry gives architecture many helps: first it teaches the use of the rule and the compass, by which (chiefly and easily) descriptions of buildings are drawn up in ground plans, and the setting out of squares, levels, and lines.

Likewise, through perspective, the lights of the sky are well led into buildings, from certain quarters of the world. By arithmetic the costs of buildings are added up, the measures are set out, and the difficult problems of symmetry are

worked through by geometric means and methods. etc.

Besides this, philosophy declares the nature of things (which in Greek is called φυσιολογία). It is necessary for an architect to have learned this with diligence, because it has many and various natural questions, especially in aqueducts. For in their courses, the turnings about, over level ground and up the rises, natural spirits or airs are generated in various ways. The hindrances they cause no man can remedy, unless he has learned from philosophy the original causes of things.

Likewise, whoever reads the books of Ctesibius or Archimedes (and of others who have written such rules) cannot think as they do, unless he has received instruction from philosophers in these things.

And he must know music, so that he can understand both regular and mathematical music, and tune his ballistas, catapults, and scorpions well. etc.

Moreover, the bronze vessels which in theaters are placed by mathematical order in niches under the steps, and the differences of the sounds (which the Greeks call ῥηεῖα) are ordered according to musical symphonies and harmonies, being distributed through the circuits by Diatessaron, Diapente, and Diapason. So that the fitting voice of the players' sound, when it came to these preparations made in order, being increased there, with that increase it might come clearer and more pleasant to the ears of the onlookers. etc.

And by astronomy are known the east, west, south, and north; the shape of the sky, the equinox, the solstice, and the course of the stars. Unless one knows these things, one cannot perceive anything at all of the reason of horologies. Since, then, this ample science is adorned, beautified, and stored with so many and various skills and knowledges, I think no one can rightly call themselves an architect all at once. Only those who, from their childhood years, rising by these degrees of knowledge, being fostered in the attainment of many languages and arts, have reached the high tabernacle of architecture. etc.

And to those to whom nature has given such quick perception, sharpness of wit, and memory, that they can be thoroughly skilled in geometry, astronomy, music, and the rest of the mathematical arts: such surpass and pass the calling and state of architects:

A mathematician.

and have become mathematicians. etc. And they are rarely found. As in times past there were Aristarchus Samius; Philolaus and Archytas, Tarentines; Apollonius Pergaeus; Eratosthenes Cyreneus; Archimedes and Scopas, Syracusians. These also left to their posterity many engines and gnomonic works, invented and explained by numbers and natural means.

All this, and the same words (in sense), in one single chapter of this incomparable architect Vitruvius, you will find. And if you were but to take his book in hand and look through it slightly, you would say at once:

Vitruvius.

This is geometry, arithmetic, astronomy, music, anthropography, hydrogogy, horometry, etc., and to conclude, the storehouse of all workmanship.

Now let us listen to our other judge, our Florentine, Leo Baptista, and look closely at how he determines of architecture. But before I go any further, he says, I think I ought to set out what man I would have allowed to be an architect. For I will not bring forward a carpenter, as though you could compare him to the chief masters of other arts. For the hand of the carpenter is the architect's instrument.

Who is an architect.

But I will define the architect as "that man who has the skill (by a certain and marvelous means and way) both in mind and imagination to determine, and also in work to finish, whatever works, by motion of weight, and coupling and framing together of bodies, may most aptly be useful for the worthiest uses of man." And to be able to perform these things, he needs the attainment and knowledge of the best and most worthy things. etc.

The whole craft of architecture in building consists in lineaments and in framing. And the whole power and skill of lineaments tends to this: that the right and absolute way may be had of fitting and joining lines and angles, by which the face of the building or frame may be comprehended and concluded. And it is the property of lineaments to prescribe to buildings, and every part of

them, a fitting place and a certain number: a worthy manner and a seemingly order: so that the whole form and figure of the building may rest in the very lineaments. etc. And we may prescribe in mind and imagination the whole forms,

The immateriality of perfect architecture.

all material stuff being kept out. This point we shall reach by marking and setting out beforehand the angles and lines, by a sure and certain direction and connection. Since, then, these things are thus:

What a lineament is.

A lineament shall be the certain and constant prescribing, conceived in mind: made in lines and angles: and finished with a learned mind and wit.

We thank you, Master Baptist, that you have so aptly brought your art, and the phrase of it, to some mathematical perfection: “by a certain order, number, form, figure, and mental symmetry;” all natural and sensible stuff set apart.

Now, then, it is evident, gentle reader, how aptly and worthily I have preferred architecture to be bred and fostered in the dominion of the peerless princess Mathematics, and to be a natural subject of hers. And the name of architecture comes from the principality which this science has above all other arts. And Plato affirms the architect to be master over all who make any work. So he is neither smith nor builder, nor separately any craftsman, but the head, the provost, the director, and judge of all artificial works and all craftsmen. For the true architect is able to teach, demonstrate, distribute, describe, and judge all works wrought. And he alone searches out the causes and reasons of all artificial things. Thus excellent is architecture: though few in our days attain to it, yet the art may not be thought of otherwise than it is in very deed worthy. Nor may we make new and imperfect definitions of ancient arts in our days, for scarcity of craftsmen: no more than we may trim down the definitions of wisdom, or honor, or of friendship or of justice. Nor will I consent to diminish in the least the perfection and dignity (by just cause) allowed to absolute architecture.

Under the direction of this art are three principal, necessary mechanical arts. Namely, housing, fortification, and shipbuilding. By housing I understand both divine service and man’s common use: public and private. Of fortification

and shipbuilding strange matter might be told you: but perhaps some will be tired with this long list already recited, and others will pick apart my rough and homely talk with you, done in a hurry, for fear you should miss this true and friendly warning, and foretaste, of mathematical power. Life is short and uncertain: times are dangerous: etc. And still the printer waiting, while my pen holds back: all these things, with further matter of ingratitude, give me occasion to pass on to the other arts remaining, with all speed possible.

Navigation



The art of Navigation demonstrates how, by the shortest good way, by the aptest direction, and in the shortest time, a sufficient ship, between any two places (in navigable passage) assigned, may be conducted: and in all storms and natural disturbances happening, how to use the best possible means, by which to recover the place first assigned. What need the master pilot has of the other arts recited here before, it is easy to know: as of hydrography, astronomy, astrol-ogy, and horometry. Presupposing continually the common base and founda-tion of all: namely arithmetic and geometry. So that he be able to understand and judge his own necessary instruments, and necessary equipment: whether they are perfectly made or not: and also can (if need be) make them himself. As quadrants, the astronomer's ring, the astronomer's staff, the universal astro-labe. A hydrographic globe. Hydrographic charts, true (not with parallel merid-ians). The common sea compass: the compass of variation: the proportional and paradoxical compasses

Year 1559.

(invented by me, for our two Muscovy master pilots, at the request of the Company) clocks with springs: hour, half-hour, and three-hour sandglasses: and various other instruments: and also be able, on a globe or a plane, to describe the paradoxical compass, and duly to use the same, for all manner of purposes to which it was invented. And also be able to calculate the planets' places for all times.

Moreover, with sun, moon, or star (or without them) be able to define the longitude and latitude of the place which he is in: so that the longitude and latitude of the place from which he sailed be given, or known to him. To which belongs an expert means of always being certain of the ship's course. etc. And by foreseeing the rising, setting, non-setting, or midnighting of certain stormy fixed stars, or their conjunctions and angles with the planets, etc., he ought to have expert conjecture of storms, tempests, and waterspouts, and such meteorological effects, dangerous at sea.

For, as Plato says, To foresee the changes and opportunities of times is fitting no less to the art of war than to husbandry and navigation.

And besides such cunning means, more evident tokens in sun and moon ought to be known by him: such as (the philosophical poet) Virgilius teaches in his *Georgics*. Where he says,

Georgic 1.

The sun also, both at his rising and when he sinks into the waves, will give signs; the surest signs follow the sun, etc.—For often we see that on his very face various colors wander. Blue, he announces rain; fiery, the east winds. But if spots begin to mingle with the ruddy fire, then you will see everything alike seething with wind and storm clouds: on such a night let no one urge me to go out over the deep, or to pull up the cable from the land, etc. The sun will give you signs. Who would dare call the sun false?—etc.

And so of moon, stars, water, air, fire, wood, stones, birds, and beasts, and of many other things, a certain sympathetic forewarning may be had: sometimes to great pleasure and profit, both on sea and land. It appears sufficiently, for my present purpose, from what has gone before, how mathematical the art of navigation is: and how it needs and also uses other mathematical arts: And now, if I were to go about to speak of the many benefits coming to this land, and others, by ships and navigation, you might think that I was seizing chances to use many words where they are not needed.

Yet this one thing I may justly say. In navigation, none ought to take greater care to be skilled than our English pilots. And perhaps some would attempt more, and others would more willingly help, if they knew for certain what priv-

ilege God had given this island, by reason of its situation, most convenient for navigation, to places most famous and rich. And though,

Year 1567 S. H. G.

(of late) a young gentleman, a courageous captain, was in great readiness, with good hope and strong grounds of persuasion, to have ventured for a discovery (either westward, by Cape de Paramantia, or eastward, above Nova Zemla and the Cyremisses), and was, at the very near time of attempting, called and employed otherwise (both then and since) in great good service to his country, as the Irish rebels have tasted:

Year 1569

Yet I say, though the same gentleman does not deal with it hereafter, some-one or other should attend to the matter: and by good advice and careful judgment, little by little, win sufficient knowledge of that trade and voyage: which now I would be sorry (through carelessness, lack of skill, and courage) should remain unknown and unheard of. Since also we are here half challenged by the learned, by a request half published. From it, truly, might grow benefit, to this land chiefly, and to the rest of the Christian commonwealth, far surpassing all riches and worldly treasure.

Thaumaturgike, 1



Thaumaturgike is the mathematical art that gives a set order for making strange works, to be perceived by the senses and to make people greatly wonder. This wonder-work is made in various ways. Some by Pneumatithmie, as the works of Ctesibius and Hero. Some by weight, of which Timæus speaks. Some by tightened strings or springs, with which living motions are imitated. Some by other means, as the images of Mercury, and the bronze head made by Albertus Magnus that seemed to speak. Boethius was excellent at these feats. To whom Cassiodorus, writing, says: Your purpose is to know deep things and to show wonders. By the arrangement of your art, metals bellow; a bronze Diomedes blows a loud trumpet; a bronze serpent hisses; made birds sing sweetly. We tell of small things of you, who can imitate heaven, etc. As for the strange self-moving work, which at Saint Denys, near Paris,

Year 1551

I saw once or twice (Orontius being then with me, in company): it would be too strange to tell. But some have written it. And yet, I hope, it is there to be seen by others. By perspective also strange things are done, as I partly gave you to understand before, in perspective: to see in the air, high up, the living image of another man, either walking back and forth or standing still. Likewise, to come into a house and there see the lively show of gold, silver, or precious

stones, and, reaching to take them in your hand, find nothing but air. By this, some men (in all other matters counted wise) have badly overshot themselves, misjudging the means. Therefore said Claudius Cælestinus.

(On those things that happen marvelously in the world. Chapter 8.)

Nowadays we see some men, of great learning and reputation, judging certain works to be marvelous, above the power of nature; of which works, one skilled in perspective could easily have given the cause. Of Archimedes' sphere, Cicero bears witness.

(Tusculan Disputations 1.)

This is very strange to think on. For when Archimedes, says he, fixed into a sphere the movements of the sun, the moon, and the five other planets, he did what the god who (in Plato's *Timæus*) made the world did: that, with one turning, he should rule motions most unlike in slowness and speed. But we have a greater cause for wonder from Claudianus' report of this, who says this work of Archimedes was made of glass, and discusses it at greater length, which I omit. The wooden dove that the mathematician Archytas made to fly is spoken of by Agellius. Plato reports the strange images of Daedalus. Homer leaves in writing Vulcan's self-movers, by hidden wheels. Aristotle mentions both in his *Politics*. The workmanship achieved in recent times by good skill in Trochilike was marvelous, etc. For in Nuremberg, an iron fly, let out of the craftsman's hand, would, as it were, fly about among the guests at the table and at last, as though weary, return to its master's hand again. Moreover, an artificial eagle was ordered to fly out of the same town a mighty way, high in the air, toward the emperor coming there, and to follow him until it reached the gate of the town.



Thus you see what mathematical art can accomplish when skill, will, industry, and ability are properly put to the proof.

A digression.

And for these, and such like marvelous acts and feats, wrought and devised naturally, mathematically, and mechanically:

Apologetic.

ought any honest student and modest Christian philosopher be counted and called a conjurer? Shall the folly of idiots and the malice of the scornful prevail so much that he who seeks no worldly gain or glory at their hands, but only from God the treasure of heavenly wisdom and knowledge of pure truth, shall he (I say) in the meantime be robbed and stripped of his good name and fame? He who seeks (by Saint Paul's advice) in the properties of creatures and their wonderful virtues just cause to glorify the eternal and almighty Creator: shall that man be condemned in secret as a companion of the hellhounds, and a caller and conjurer of wicked and damned spirits? He who mourns his great lack of time, enough (to his satisfaction) to learn godly wisdom and godly truths in, and sets all his delight only in that: will that man lose and misuse his time in dealing with the chief enemy of Christ our Redeemer, the deadly foe of all humankind, the subtle and shameless perverter of godly truth, the hypocritical crocodile, the envious basilisk, forever desiring, in the blink of an eye, to destroy all humankind, both body and soul, for eternity? Surely, for my part, having something to say here, I have not learned to make so brutish and so wicked a bargain. Should I, for my twenty or twenty-five years of study, for two or three thousand marks spent, for seven or eight thousand miles of going and traveling, only for the sake of good learning, and that in all manner of weather, in all manner of ways and passages, early and late, in danger of violence from people, in danger of destruction by wild beasts, in hunger, in thirst, in perilous heat by day with toil on foot, in dangerous damp cold by night almost taking away life (as God knows), with lodgings often of little comfort and sometimes of less safety? And for much more than all this done and suffered for learning and gaining wisdom, should I (I pray you), for all this, no otherwise nor more warily, or by God's mercy any more luckily, have fished with so large and costly a net, so long in the drawing (and that with the help and advice of Lady Philosophy and Queen Theology), only at length to have caught and drawn up a frog?

A proverb: fair fished, and caught a frog.

Or a devil? For so the common peevish babbler imagines and jangles, and so the malicious scorner secretly wishes, and brazenly faces me down behind my back. Ah, what a miserable thing is this kind of people! How great is the blindness and boldness of the crowd in things above their capacity! What a land, what a people, what manners, what times are these! Have they become devils themselves, and by bearing false witness against their neighbor would they also become murderers? Does God give them so long a respite, to pull themselves back from this horrible slandering of the guiltless, contrary to their own consciences, and still will they not stop? Does the innocent person forbear calling them to answer him legally, according to the rigor of the laws, and will they despise his charitable patience? As they, against him by name, forge, fable, rage, and raise slander by word and print, will they provoke him also, by word and print, to set their names before the world, with their particular devices, fables, beastly imaginations, and un-Christian slanders? Well, well. O you who are such, my unkind countrymen. O unnatural countrymen. O ungrateful countrymen. O brainsick, rash, spiteful, and disdainful countrymen. Why do you oppress me so violently with your slandering of me, contrary to truth and contrary to your own consciences? And I, to this hour, neither by word, deed, or thought, have in any way been harmful, damaging, or wrong to you or yours? Have I, for so long, so dearly, so far, so carefully, so painfully, so dangerously sought and traveled for the learning of wisdom and the gaining of virtue, and in the end (in your judgment) have I become worse than when I began? Worse than a madman? A dangerous member of the commonwealth, and no member of the church of Christ? Do you call this being learned? Do you call this being a philosopher and a lover of wisdom? To leave the straight heavenly way and wallow in the broad way of damnation? To leave the light of heavenly wisdom and lurk in the dungeon of the prince of darkness? To leave the truth of God and his creatures and fawn on the shameless, crafty, stubborn liar who disgraces God's truth to the uttermost of his power? To leave eternal life and bliss and cling to the author of everlasting death, that murderous tyrant most greedily awaiting the prey of a person's soul? Well, I thank God and our Lord Jesus Christ for the comfort I have from the examples of other men before my time,

to whom I am not worthy to be compared, either in godliness of life or in perfection of learning; and yet they sustained very like injuries to those I do, or rather greater. Patient Socrates' Apology will testify; Apuleius' Apologies will declare the brutishness of the crowd. Ioannes Picus, Earl of Mirandula, his Apology will show you the raging slander of the malicious and ignorant against him. Ioannes Trithemius' Apology will set out how he had occasion to make a public protestation, both because of the rude and simple and also in regard to such as were counted of the wisest sort of men. "Many could I recite: but I put off the precise and settled handling of this matter, being reluctant to expose the folly and malice of my native countrymen.



who can hardly digest or like any extraordinary course of philosophical studies that does not fall within the compass of their capacity, or where they are not let in on the true and secret cause of such wonderful philosophical feats." These men are of four sorts, chiefly. The first I may name: vain, prating busybodies. The second: fond friends. The third: imperfectly zealous. And the fourth: malicious ignorants. To each of these, briefly and in charity, I will say a word or two, and so return to my preface.

(1.)

Vain, prating busybodies: keep your idle gatherings and discussions to something other than talk of matters that are either above your capacity for difficulty or contrary to your conscience in truth.

(2.)

Fond friends, stop commending a friend you hardly know out of blind affection, on the grounds that because he knows more than the ordinary student he must therefore be skilled and a doer in such matter and manner as you call conjuring, thinking that by that you advance his fame, and that you make others greatly marvel at your luck in having such a learned friend. Stop attributing impiety where you pretend friendship. For if your tongues were true, then that friend of yours would be untrue both to God and to his sovereign. Such friends and fondlings: I shake you off and renounce you. Shake off your folly.

(3.)

To you, imperfectly zealous, I say: perhaps you mean well, but you miss the mark, if you will kill a lamb to feed the flock with its blood. Sheep have no natural nourishment from lamb's blood, and no more is Christ's flock properly built up by horrible slanders. Nor is your fair pretense at all well graced by such rough, ragged rhetoric. But those who use me so will find a foul crack in their credit. Speak what you know, and know as you ought: do not know by hearsay when life lies in danger. Search to the quick, and let charity be your guide.

(4.)

Malicious ignorants, what shall I say to you? Keep your tongue from evil. Keep your tongue from slander. Though your tongues are sharpened like serpents, and adder's poison lies in your lips:

(Psalm 140.)

yet take heed, and think in time to yourself: A man of loose tongue shall not be established in the land. Evil shall hunt the violent man until he is cast down. For I am sure that the Lord will do justice for the afflicted, and vengeance for the poor.

Thus I ask you, my trusted friends and countrymen (you mathematicians, mechanicians, and philosophers, charitable and discreet), to deal on my behalf with the frivolous and untrue-tongued, my envious adversaries or fond friends. Further, I would wish that at leisure you would consider how Basilius Magnus lays Moses and Daniel before the eyes of those who count all such philosophical studies as mine have been ungodly or unprofitable. Weigh well Saint Stephen's witness of Moses.

(Act. 7. C.)

Moses was instructed in all manner of wisdom of the Egyptians, and he was powerful both in his words and in his works. You see that this philosophical power and wisdom which Moses had was in no way disapproved by the Holy Ghost. Yet Plinius has recorded Moses to be a wicked magician. And that must of necessity be either for this philosophical wisdom learned before his calling to lead the children of Israel, or for those his wonders worked before King Pharaoh after he had the conducting of the Israelites. As for the first, you perceive how Saint Stephen, at his martyrdom (being full of the Holy Ghost), in

his recapitulation of the Old Testament made mention of Moses' philosophy with full approval of it, and Basilius Magnus too attests that it was profitable to Moses (and therefore, I say, necessary to the church of God). But as for Moses' wonders done before King Pharaoh: God himself said: See that you do all those wonders before Pharaoh which I have put in your hand. Thus you clearly perceive how rashly Plinius has slandered Moses,

(Book 30, chapter 1.)

of vain and fraudulent magic, saying: There is also another sect of magic, depending on Moses, Jannes, and Iotape, and the Jews, but many thousands of years after Zoroaster. etc.

(1.)

Let all such, therefore, who are far inferior to Plinie in judgment and skill of philosophy, "take good heed, for fear that they overshoot themselves rashly," in



judging of philosophers' strange acts, and the means by which they are done.

Thaumaturgike, 2



(2.)

But much more ought they to beware of forging, devising, and imagining monstrous feats and wonderful works where and when no such were done: not even any spark or likelihood of such things as they, without any shame, report.

(3.)

And to conclude, most of all let them be ashamed before humankind, and afraid of the dreadful and just judge: both to devise foolishly or maliciously, and then devilishly to father their new-fangled monsters on me, innocent in hand and heart, for trespassing against the law of God or of humankind in any of my studies or exercises, philosophical or mathematical, as in due time I hope will be more evident.

Archemastrie



Now I end with Archemastrie. The name is not so new as this art is rare. For another art, a degree below it in skill and power, has already been given this English name. And yet this may serve our purpose well enough at present. This art teaches how to bring to actual sensible experience all worthy conclusions proposed by all the mathematical arts and concluded by true natural philosophy; and it both adds to them a further scope in the terms of the same arts, and also by its own method, in peculiar terms, proceeds with the help of the said arts to the performance of complete experiences which cannot (formally) be challenged from any particular art. If you remember how we considered architecture in respect of all common handiwork, you may get some light from that toward understanding the sovereignty and property of this science. I may call it a science rather than an art, for the excellence and mastery it has over so many and so mighty arts and sciences. And because it proceeds by experiences, and searches out the causes of conclusions by experiences, and also puts the conclusions themselves into experience, some name it the experimental science. Nicolaus Cusanus calls it so in his *Experimentes Statikall*, and another philosopher,

native to this land (the flower of whose worthy fame can never die or wither) wrote about it at length at the request of Clement the Sixth. The art carries with it wonderful credit, because it certifies sensibly, fully, and completely, to

the utmost power of nature and art. This art certifies by complete and absolute experience; other arts, with their arguments and demonstrations, persuade, and in words prove their conclusions very well.



But words and arguments are no sensible certifying, nor the full and final fruit of sciences that can be practiced. And though some arts have experiences in them, they are not complete, nor brought to the uttermost they may be stretched to, and applied sensibly. For example: the natural philosopher disputes and makes a goodly show of reason, and the astronomer and the optical mechanician put some things into experience, but neither all that they can, nor yet fully and to the utmost those they do. There the Archemaster steps in and leads the experiences on, by order of his experimental doctrine, to the chief and final power of natural and mathematical arts. Of two or three men in whom this description of Archemastery was verified in practice I have read and heard, and good record exists of such perfection in them. So this art is no fantastic imagination, as some sophister might flourish with his insolubles, dazzle your imagination, and dash your honest desire and courage from believing these things so unheard-of, so marvelous, and of such importance. Well, as you will. I have forewarned you. I have done the part of a friend; I have discharged my duty toward God for the small talent I received from his most merciful hands. To this science the science Alnirangiat does great service. Do not wonder at this name: I do not change the name so used and published in print by others, being a name proper to the science. Under this comes *Ars Sintrillia* [title unclear in the source], by Artephius, briefly written. But the chief science of the Archemaster (in this world) as yet known is another science, as it were, optical; its name shall be told (God willing) when I have some more just occasion to discourse of it.

Here I must end, thus abruptly (gentle friend, and unfeigned lover of honest and necessary truths). For those who have (for your sake and virtue's sake) requested me (an old, worn-out mathematician) to take pen in hand, through the confidence they placed in my long experience and tried sincerity, for the declaring and reporting somewhat of the fruit and benefit to be attained by the

mathematical arts: even they, very much against their own wills, are forced for various reasons to satisfy the workman's request by ending at once. He fears so new an attempt and so costly, and a matter so little (so far) considered or esteemed among the common sort of students.

And whereas I was pressed to allege some reason why, in our common speech, this part of the principal science of geometry, called Euclid's Geometric Elements, is published for you to handle, being people without Latin and not university scholars: truly I think it needless.

(1.)

For the honor and standing of the universities and their graduates is in no way diminished by this, seeing that you receive all this benefit from and through their nurslings, however great it be.

(2.)

Neither are their studies in any way hindered by it, any more than the Italian universities, as the academies of Bologna, Ferrara, Florence, Milan, Padua, Pavia, Perugia, Pisa, Rome, and Siena, or any one of them, find themselves at all disgraced or their studies hindered in the least by Friar Lucas de Burgo or by Nicolaus Tartalea, who in the common Italian language have published not only Euclid's geometry but something of Archimedes, and in arithmetic and practical geometry very large volumes, all in their common speech. Nor have the famous universities in Germany been at all displeased with the geometric institutions of Albertus Durerus in Dutch; nor with the learned translation by Gulielmus Xylander of the first six books of Euclid from Greek into high Dutch; nor with the geometric volume of Gualterus H. Riffius, very diligently translated into the high Dutch tongue and published. Nor do the universities of Spain or Portugal think their reputation decayed, or suppose any of their studies hindered by the excellent mathematical works of P. Nonnius put forth in common speech. Do you not likewise have in French the whole mathematical quadrivium? And yet neither Paris, Orleans, nor any of the other universities of France has at any time been offended with the translators or publishers, nor has any person's study been hindered by it?

(3.)

And surely the common and ordinary scholar (and much more the grammarian) before coming to the university can (or may) now be instructed well enough (according to Plato's counsel) in arithmetic and geometry, for the better and easier learning of all manner of philosophy, academic or Peripatetic. And by that means go on more cheerfully, more skillfully, and more quickly in his studies there to be learned, and so profit more in less time than otherwise he should or could do.

(4.)

Also many good and fertile English wits, of young gentlemen and of others who never intend to meddle with the profound search and study of philosophy (to be learned in the universities), may nevertheless now, with more ease and liberty, have good occasion to put the sharpness of their wits to virtuous use; where otherwise they might (perhaps) spend their time in foolish exercises, or rather lose it, serving neither God nor furthering the common or private good.

(5.)

And great comfort with good hope may the universities have from this English geometry and mathematical preface, that they hereafter shall be more regarded, esteemed, and resorted to. For when it shall be known and reported that from the mathematical sciences only such great benefits follow as I have specified, and that indeed some of you students without Latin can be good witnesses of such rare fruit enjoyed by you by that, as either was not heard of before or else not so fully credited: "Well, may all men conjecture that far greater aid and better furniture to win to the perfection of all philosophy,

Universities.

may be had in the universities, being the storehouses and treasury of all sciences,



and all arts necessary for the best and most noble state of commonwealths."

(6.)

Besides this, how many a common craftsman is there in these realms of England and Ireland that deals with numbers, rule, and compass, who, with their own skill and experience already had, will be able (by these good helps and

information) to find out and devise new works, strange engines, and instruments: for various purposes in the commonwealth, or for private pleasure, and for the better maintaining of their own estate? I will not (therefore) fight against my own shadow. For no man, I am sure, will open his mouth against this enterprise; no one, I say, who either has charity toward his brother (and would be glad to further him in virtuous knowledge), or who has any care and zeal for the bettering of the common state of this realm; neither any who make account of what the wiser sort of men (sage and steady) think of them. To none, therefore, will I make any apology for doing a virtuous act, and for commending or setting forth profitable arts to English people in the English tongue. “But to God our Creator let us all be thankful: that as he, of his goodness, by his power, and in his wisdom,



has created all things in number, weight, and measure; so to us, of his great mercy, he has revealed means by which to attain the sufficient and necessary knowledge of the said three principal instruments of his: which means I have abundantly proved to you to be the sciences and mathematical arts.”

And though I have been pressed by the straightness of time, so that in no way could I set the matter down (in my mind) as I determined, hoping for convenient leisure: yet if virtuous zeal and honest intent provoke and bring you to the reading and examining of this compendious treatise, I do not doubt but that, as the truth of it (according to our purpose) will be evident to you, so its pith and force will persuade you, and its wonderful fruit highly please you. And that you may the more easily perceive and better remember the principal points my preface treats of,

The ground plan of this preface in a table.

I will give you the ground plan of my whole discourse in an annexed table, from the first to the last, somewhat methodically devised.

If haste has made my poor pen stumble anywhere, you will (I am sure) partly repay me, for my earnest and sincere good will to please you, by considering the rocky, huge mountains and the perilous, untrodden ways it has toiled

and labored through (night and day the while) to bring you this good news and comfortable proof of virtue's fruit.

So I commit you to God's merciful direction for the rest, heartily beseeching him to prosper your studies and honest intents, to his glory and the benefit of our country. Amen.

Written at my poor house at Mortlake.

In the year 1570. February 9.

J. DEE

The Groundplat of the Mathematical Praeface



Here you have (according to my promise) the ground plan of my mathematical preface, annexed to Euclid, now first published in our English tongue. An. 1570. Febr. 3.

Sciences and mathematical arts are either,
Principal, which are two only,
Arithmetic.

Simple, which deals with numbers only, and demonstrates all their properties and appurtenances, where a unit is indivisible. Mixed, which with the help of principal geometry demonstrates some arithmetic conclusion or purpose.

Geometry.

Simple, which deals with magnitudes only, and demonstrates all their properties, affections, and appurtenances, whose point is indivisible. Mixed, which with the help of principal arithmetic demonstrates some geometric purpose, as Euclid's Elements.

The use of which is either,

In supernatural, eternal, and divine things: by application, ascending. In mathematical things: without further application. In natural things: both substantial and accidental, visible and invisible, etc. By application, descending.

The like uses and applications are (though a degree lower) in the derived mathematical arts.

Derived from the principals; of which some have

The names of the principals, as:

Common arithmetic, which considers

—Arithmetic of the most usual whole numbers, and of the fractions belonging to them. —Arithmetic of proportions. —Circular arithmetic. —Arithmetic of radical numbers: simple, compound, mixed, and of their fractions. —Arithmetic of cossic numbers, with their fractions, and the great art of Algiebar.

Common geometry, which teaches measuring

At hand

All lengths—mecometry. All planes, as land, board, glass, etc.—embadome-try. All solids, as timber, stone, vessels, etc.—stereometry.

With distance from the thing measured, as,

How far from the measurer anything is, seen by him on land or water, called apomecometry. How high or deep, from the level of the measurer's standing, anything is, seen by him on land or water, called hypsometry. How broad a thing is that lies in the measurer's view, as long as it is situated on land or water, called platometry.

From which have grown the feats and arts of

Geodesy: to measure and survey lands, woods, waters more skillfully, etc. Geography. Chorography. Hydrography. Stratarithmetrie.

Proper names, as:

Perspective,—which demonstrates the ways and properties of all radiations: direct, broken, and reflected.

Astronomy,—which demonstrates the distances, magnitudes, and all natural motions, appearances, and affections proper to the planets and fixed stars, for any time past, present, and to come, with respect to a certain horizon or without respect to any horizon.

Music,—which demonstrates by reason and teaches by sense how to judge and order perfectly the diversity of sounds, high or low.

Cosmography,—which wholly and perfectly describes the heavenly and also the elemental part of the world, and of these parts makes the homologous application and mutual comparison that are necessary.

Astrology,—which reasonably demonstrates the operations and effects of the natural beams of light and the secret influence of the planets and fixed stars in every element and elemental body, at all times, in any horizon assigned.

Statics,—which demonstrates the causes of heaviness and lightness in all things, and the motions and properties belonging to heaviness and lightness.

Anthropography, which describes the number, measure, weight, figure, situation, and color of every diverse thing contained in the perfect body of man, and gives certain knowledge of the figure, symmetry, weight, characterization, and due local motion of any part of the said body assigned, and of the numbers belonging to the said part.

Trochilike,—which demonstrates the properties of all circular motions: simple and compound.

Helicosophy,—which demonstrates the drawing of all spiral lines: in a plane, on a cylinder, cone, sphere, conoid, and spheroid, and their properties.

Pneumatithmie,—which demonstrates by close hollow geometric figures (regular and irregular) the strange properties (in motion or rest) of water, air, smoke, and fire in their continuity, and as they are joined to the elements next them.

Menadrie,—which demonstrates how, above nature's virtue and simple power, virtue and force can be multiplied; and so to direct, to lift, to pull toward, and to push or cast from, any multiplied or simple determined virtue, weight, or force that is not naturally so directable or movable.

Hypogeiodie,—which demonstrates how, under the spherical surface of the earth, at any depth, to any perpendicular line assigned (whose distance from the perpendicular of the entrance, and the azimuth likewise with respect to the said entrance, being known), a certain way may be laid out and traveled, etc.

Hydragogie,—which demonstrates the possible leading of water, by nature's law and by artificial help, from any head (being spring, standing, or running water) to any other place assigned.

Horometry,—which demonstrates how, at all appointed times, the precise usual denomination of time may be known for any place assigned.

Painting,—which demonstrates and teaches how the intersection of all visual pyramids, made by any plane assigned (the center, distance, and lights being determined), may be represented by lines and proper colors.

Architecture,—which is a science adorned with many doctrines and diverse instructions, by whose judgment all works finished by other workmen are judged.

Navigation,—which demonstrates how, by the shortest good way, by the aptest direction, and in the shortest time, a sufficient ship between any two places (in navigable passage) assigned may be conducted; and in all storms and natural disturbances happening, how to use the best possible means to recover the place first assigned.

Thaumaturgike,—which gives a set order for making strange works, to be perceived by the senses, and to make people greatly wonder.

Archemastrie,—which teaches how to bring to actual sensible experience all worthy conclusions proposed by all the mathematical arts and concluded by true natural philosophy; and it both adds to them a further scope in the terms of the same arts, and also by its proper method, in peculiar terms, proceeds with the help of the said arts to the performance of complete experiences, which cannot (formally) be challenged from any particular art.

Imprinted by John Day.

An. 1570. Feb. 25.