



The Mathematicall Praeface to the Elements of Geometrie

JOHN DEE

Musean Translation

MOUSEIA



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Translated by Michael Berardi

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THIS EDITION

Modern machine-assisted Musean edition by Mouseia, from Dee's 1570 Praeface. Latin quotations and titles are translated into English, using Dee's own English glosses where provided. Printer's type-size markup is normalized; marginal notes are kept as asides. Billingsley's separate address is omitted. Uncertain readings are marked.

TRANSLATOR

Michael Berardi

THE WORK

Written 1570 · Treatise on the mathematical sciences

SOURCE

Original text from gutenberg.org.

GAPS AND UNCERTAINTIES

The 1570 printing's oversized fold-out Groundplat is reproduced in the source transcription as a plain list; its arrangement is the transcriber's. Greek words and quotations are kept as printed. Latin quotations are not kept in the English editions; the astronomical term *primum mobile* is kept as a conventional technical term, and *Geometria* is kept where Dee discusses the word itself. Dee's printed Latin has irregular forms, so a few readings are uncertain: the name printed *Iamne* in the Pliny sentence is read as *Jannes*, and the title *Ars Sintrillia* attributed to *Artephius* is kept as printed because its meaning and source are unclear.

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To the Lovers of Truth and Students of Noble Sciences



To the unfeigned lovers of truth, and the constant students of the noble sciences, John Dee of London heartily wishes grace from heaven, and the most prosperous success in all their honorable attempts and exercises.

Divine Plato, that great master of many worthy philosophers, the steady affirmer and vigorous persuader of the One, the Good, and Being, was visited in his school and academy at various times — besides his ordinary scholars — by a certain kind of men, allured by the noble fame of Plato and the great commendation of his profound and profitable doctrine. But when such hearers, after long listening to him, perceived that the drift of his discourses came out to conclude that the One, the Good, and Being are spiritual, infinite, eternal, omnipotent, etc., nothing being alleged or expressed as to how worldly goods, how worldly dignity, how health, strength, or vigor of body might be attained, nor yet by what means a marvelous sensible and bodily bliss and felicity hereafter might be reached — at once the fancies of those hearers were dampened; their opinion of Plato was completely changed; indeed his doctrine was despised by them, and his school visited by them no more. Which thing his scholar Aristotle, narrowly considering, found the cause of it to be “that they had no forewarning and general information as to where his doctrine tended.” For so they might have had occasion either to refrain from haunting his

school — if they then disliked his aim and purpose — or to continue in it constantly, to their full satisfaction, if such his final scope and intent had been to their desire. For this reason Aristotle, ever after, used briefly to forewarn his own scholars and hearers “both of what matter, and also to what end, he undertook to speak or teach.”

While I consider the diverse methods of these two excellent philosophers (and am most sure both that Plato could very well have taught otherwise, and that Aristotle might have dealt boldly with his hearers in the same way as Plato did), I am in no small pang of perplexity: because that which I dislike is easiest for me to perform (and to have Plato for my example), and that which I know to be most commendable, and (in this first bringing into common handling the mathematical arts) most necessary, is full of great difficulty and sundry dangers. Yet neither do I think it fitting, for so strange a matter as is now meant to be published, and to so strange an audience, to be bluntly at first put forth without a special preface; nor (imitating Aristotle) can I well hope that, according to the breadth and dignity of the mathematical discipline, I am able either plainly to set out the bounds of its subject matter, or precisely to express its chief purposes and most wonderful applications. And though I am sure that such as shrank from Plato’s school, after they had perceived his final conclusion, would in these things have been his most diligent hearers — so infinitely might their desires in the end and at length be satisfied by our mathematical arts — yet by this my preface and forewarning, all such may, to their great benefit, be the sooner lured here; and also the Pythagorean and Platonic perfect scholar, and the constant profound philosopher, with more ease and speed may, like the bee, gather from this both wax and honey.

(The intent of this preface.)

Therefore, since I find great occasion — for the causes alleged, and further in respect of my art of mathematics in general — to use “a certain forewarning and preface, whose content shall be that mighty, most pleasant, and fruitful mathematical tree, with its main arms and second grafted branches: what each one of them is, and also what benefit in general is to be looked for, both from graft and from stock.” And since this enterprise is so great that, to our time, it

has never (to my knowledge) been achieved by anyone; and also it is most hard in these dreary days to win due and common credit for such rare and strange arts — nevertheless, if for my sincere endeavor to satisfy your honest expectation you will lend me your thankful mind a while, and apply your eye or ear attentively to such matter as my pen is able to deliver this time with speed, perhaps at once, and for the first saluting, you will find this preface a long enough lesson. And either you will by this be made much the apter for a second, or shortly become well able yourselves, from the lion's claw, to conjecture his royal symmetry and further property. Now then, gentle friends and countrymen, turn your eyes and bend your minds to that doctrine which for our present purpose my simple talent is able to yield you.

All things that are, and have being, are found under a triple general diversity. For either they are deemed supernatural, natural, or of a third being. Supernatural things are immaterial, simple, indivisible, incorruptible, and unchangeable. Natural things are material, compounded, divisible, corruptible, and changeable. Supernatural things are comprehended by the mind alone; natural things are able to be perceived by the outward sense. In natural things probability and conjecture have a place; but in supernatural things chief demonstration and most sure knowledge is to be had. By which properties and comparisons of these two, the state, condition, nature, and property of those things which we earlier called "of a third being" may more easily be described; which also, by a special name, are called Mathematical Things. For these, being in a manner middle between supernatural and natural things, are not so absolute and excellent as supernatural things, nor yet so base and gross as natural things; but they are immaterial, and nevertheless by material things able to be somewhat signified. And though their particular images by art are aggregable and divisible, yet the general forms notwithstanding remain constant, unchangeable, untransformable, and incorruptible. Neither can they at any time be perceived or judged by the sense. Nor yet for all that are they first conceived in the royal mind of man. But surmounting the imperfection of conjecture, supposition, and opinion, and falling short of high intellectual conception, they are the mercurial fruit of discursive reasoning, subsisting in perfect

imagination. A marvelous neutrality have these mathematical things, and also a strange participation between supernatural, immortal, intellectual, simple and indivisible things, and natural, mortal, sensible, compounded and divisible things. Probability and sensible prose may well serve in natural things, and is commendable; in mathematical reasonings a probable argument is nothing regarded, nor yet the testimony of sense at all credited; but only a perfect demonstration of truths certain, necessary, and invincible, universally and necessarily concluded, is allowed as sufficient for “an argument exactly and purely mathematical.”

(Note the word “unit” to express the Greek Monas, and not “unity,” as we have all commonly used until now.)

Of mathematical things there are two principal kinds: namely, Number and Magnitude.

(Number.)

Number, we define, to be a certain mathematical sum of units. And a unit is that mathematical thing, indivisible, by sharing in some likeness of whose property any thing which in deed or in count is One may reasonably be called One. We account a unit a mathematical thing, though it is no number and also indivisible, because number materially consists of it, which principally is a mathematical thing.

(Magnitude.)

Magnitude is a mathematical thing, by sharing in some likeness of whose nature any thing is judged long, broad, or thick. “A thick magnitude we call a solid, or a body. What magnitude soever is solid or thick is also broad and long. A broad magnitude we call a surface or a plane. Every plane magnitude has also length. A long magnitude we call a line. A line is neither thick nor broad, but only long; every certain line has two ends:

(A point.)

The ends of a line are called points. A point is a mathematical thing, indivisible, which may have a certain determined position.” If a point moves from a determined position, the way in which it moved is also a line, mathematically produced; after which, of the ancient mathematicians,

(A line.)

a line is called the race or course of a point. A point we define by the name of a mathematical thing, though it is no magnitude and is indivisible, because it is the proper end and bound of a line, which is a true magnitude.

(Magnitude.)

And magnitude we may define to be that mathematical thing which is divisible forever into divisible parts — long, broad, or thick. Therefore though a point is no magnitude, yet terminatively we reckon it a mathematical thing (as I said) by reason it is properly the end and bound of a line. Neither number nor magnitude has any materiality.

Of Number and Magnitude



First we will consider of number, and of the science mathematical appropriate to it, called Arithmetic: and afterward of magnitude, and its science, called geometry. But that name does not content me; of which a word or two shall be said hereafter. How immaterial and free from all matter Number is, who does not perceive? Indeed, who does not wonder at it greatly? For neither pure Element, nor Aristotle's Fifth Essence, is able to serve as the proper matter of Number. Nor yet will the purity and simplicity of Spiritual or Angelic Substance be found proper enough for it. And therefore the great and godly philosopher Anitius Boetius said: All things (which from the very first original being of things have been framed and made) do appear to be formed by the reason of Numbers. For this was the principal example or pattern in the mind of the Creator. O comfortable allurements, O ravishing persuasion, to deal with a Science whose Subject is so Ancient, so pure, so excellent, so surpassing all creatures, so used by the Almighty and incomprehensible wisdom of the Creator in the distinct creation of all creatures: in all their distinct parts, properties, natures, and virtues, by order and most absolute number, brought from Nothing to the formality of their being and state. By the property of numbers, therefore, learned by us by all possible means (to the perfection of the Science), we may both wind and draw ourselves into the inward and deep search and view of all creatures' distinct virtues, natures, properties, and forms; and also arise further, climb, ascend, and mount up (with speculative wings) in spirit to

see in the Glass of Creation the Form of Forms, the Exemplar Number of all things numerable, both visible and invisible, mortal and immortal, corporeal and spiritual. Part of this profound and divine Science had Joachim the prophet attained: by Formal, Natural, and Rational Numbers, foreseeing, concluding, and foretelling great particular events long before their coming. His books yet remaining are good proof of this; and the noble Earl of Mirandula, besides that, a sufficient witness that Joachim in his prophecies proceeded by no other way than by Formal Numbers. And this Earl himself, in Rome, (A.D. 1488.)

* set up 900 Conclusions, in all kinds of sciences, openly to be disputed; and among the rest, in his Mathematical Conclusions (in the eleventh Conclusion) he has in Latin this English sentence: By Numbers, a way is had to the searching out and understanding of every thing able to be known. For the verifying of which Conclusion, I promise to answer the 74 questions written below by the way of Numbers. Which Conclusions I omit here to rehearse: as well avoiding superfluous prolixity, as because the works of Joannes Picus are commonly available. But in any case I would wish that those Conclusions were read diligently and understood by such as are earnest observers and considerers of the constant law of numbers, which is planted in natural and supernatural things, and is prescribed to all creatures inviolably to be kept. For so, besides many other things in those Conclusions to be marked, it would appear how sincerely, and within my bounds, I disclose the wonderful mysteries to be attained by numbers.

Of my former words it is easy to gather that Number has a triple state: one in the Creator; another in every creature (in respect of its complete constitution); and the third in Spiritual and Angelic Minds, and in the Soul of man. In the first and third state Number is termed Number Numbering. But in all creatures otherwise, Number is termed Number Numbered. And in our Soul Number bears such sway, and has such affinity with it, that some of the old philosophers taught the human Soul to be a self-moving Number. And indeed in us, though it is a mere accident, yet such an accident is it that before all creatures it had perfect being in the Creator, eternally. Number Numbering, there-

fore, is the discretion that discerns and distinguishes things. But in God the Creator this discretion in the beginning produced all things orderly and distinctly. For his Numbering then was his Creating of all things. And his continual Numbering of all things is the conservation of them in being; and where and when he will lack a Unit, there and then that particular thing shall be discreated. Here I stop. But our separating, distinguishing, and Numbering creates nothing; of Multitude considered it makes a certain and distinct determination. And although these things are weighty and truths of great importance, yet (by the infinite goodness of the Almighty Ternary) artificial methods and easy ways are made by which the zealous philosopher may come near this Ida of many rivers, this Mountain of Contemplation, and more than Contemplation. And also, though Number be a thing so immaterial, so divine, and eternal, yet by degrees, little by little, stretching forth and applying some likeness of it — first to spiritual things, then bringing it lower to things sensibly perceived, as of a momentary sound repeated, then to the least things that may be seen, numerable, and at length (most grossly) to a multitude of any corporeal things seen or felt — and so from these gross and sensible things we are trained to learn a certain image or likeness of numbers, and to use art in them to our pleasure and profit. So gross is our conversation, and dull our apprehension, while mortal Sense in us rules the commonwealth of our little world. By this we say, Three lions are three, or a ternary. Three eagles are three, or a ternary.

(☞)

Which * ternaries are each the union, knot, and uniformity of three discrete and distinct units. That is, we may in each ternary thrice separately point out and show a part: One, One, and One. Where in numbering we say One, two, Three. But how far these visible Ones differ from our indivisible Units (principally considered in pure arithmetic) no one is ignorant. Yet from these gross and material things may we be led upward by degrees, so informing our rude imagination toward the conceiving of Numbers absolutely — not supposing, nor admixing any created thing, corporeal or spiritual, to support, contain, or

represent those Numbers imagined — that at length we may be able to find the number of our own name gloriously exemplified and registered in the book of the Trinity most blessed and eternal.

But further understand that vulgar practitioners have Numbers otherwise in sundry considerations, and extend their name further than to Numbers whose least part is a Unit. For the common accountant, reckoning master, or arithmetician, in his using of Numbers, imagines lesser parts of a Unit, and calls them Fractions. As of a Unit he makes a half, and thus notes it, $\frac{1}{2}$; and so of other (infinitely diverse) parts of a Unit. Indeed he further has fractions of fractions, etc.

Arithmetic, 1



And since Addition, Subtraction, Multiplication, Division, and Extraction of Roots are the chief and sufficient parts of arithmetic: which is the science that demonstrates the properties of numbers, and all operations to be performed in numbers:

“How often, therefore, these five sundry sorts of operations differ, for the most part in their execution, from the five operations of like general property and name practicable in our whole numbers, so often (for a more distinct doctrine) we commonly account and name it another kind of arithmetic.” And by this reason:

(1.)

the consideration, doctrine, and working in whole numbers only, where of a Unit no lesser part is to be allowed, is named (as it were) an arithmetic by itself. And so of the arithmetic of fractions.

(2.)

In like manner the necessary, wonderful, and secret doctrine of Proportion and proportionality has won for itself a special manner of handling and working, and so may seem another form of arithmetic.

(3.)

Moreover, astronomers, for speed and more convenient calculation, have devised a special manner of ordering numbers about their circular motions by sixties and sixtieths — by Signs, Degrees, and Minutes, etc. — which com-

monly is called the arithmetic of astronomical or physical fractions. That have I briefly noted by the name of Circular Arithmetic, because it is also used in circles that are not astronomical, etc.

(4.)

Practice has led Numbers further, and has framed them to take on the show of magnitude's property, which is incommensurability and irrationality. (For in pure arithmetic a Unit is the common measure of all numbers.) And here numbers are become like lines, planes, and solids: sometimes rational, sometimes irrational. And they have proper and peculiar characters (as $^2\sqrt{}$, $^3\sqrt{}$, and so of others, which is to signify square root, cubic root, and so forth), and proper and peculiar fashions in the five principal parts; therefore the practitioner esteems this a diverse arithmetic from the other. Practice brings in here diverse compounding of numbers: as sometimes two, three, four (or more) radical numbers diversely knit by signs of More and Less, as thus $^2\sqrt{12} \ ^3\sqrt{15}$; or thus $^4\sqrt{19} \ ^3\sqrt{12} - ^2\sqrt{2}$, etc.; and sometimes with whole numbers, or fractions of whole numbers, among them: as $20 \ ^2\sqrt{24}$, $^3\sqrt{16}$, $33 - ^2\sqrt{10}$, $^4\sqrt{44}$, $12\frac{1}{4}$, $^3\sqrt{9}$. And so infinitely may the variety happen. After this, both the one and the other have fractions incident, and so is this arithmetic greatly enlarged by the diverse exhibiting and use of compositions and mixtures. Consider how I, being desirous to deliver the student from error and cavillation, give to this practice the name of the arithmetic of radical numbers — not of irrational or surd numbers, which are at times rational, though they have the sign of a root before them, which the arithmetic of whole numbers, most usual, would say they had no such root of, and so account them surd numbers; which generally spoken is untrue, as Euclid's tenth book may teach you. Therefore to call them generally radical numbers (by reason of the sign $\sqrt{}$ prefixed) is a sure way, and a sufficient general distinction from all other ordering and use of numbers. And yet besides all this, consider the infinite desire of knowledge and incredible power of man's search and capacity: how they jointly have waded further (by mixing speculation and practice), and have found out and attained the very chief perfection (almost) of the practical use of numbers. Which thing is well to be perceived in that great arithmetic art of equation, commonly called the Rule

of Coss, or Algebra. The Latins termed it the Rule of the Thing and its Value; an apt name, comprehending the first and last points of the work. And the common names, both in Italian, French, and Spanish, depend (in naming it) upon the meaning of the Latin word for “a thing” — unless they use the name Algebra. And in this (commonly) is a double error: the one of them that think it to be of Geber’s inventing; the other of such as call it Algebra. For first, though Geber, for his great skill in numbers, geometry, astronomy, and other marvelous arts, might have seemed able to have first devised the said rule, and also the name carries with it a very near likeness to Geber’s name, yet it is true that a Greek philosopher and mathematician named Diophantus, before Geber’s time, wrote 13 books of it (of which six are yet extant; and I had them for use,

(* A.D. 1550.)

of the famous mathematician, and my great friend, Petrus Montaneus:) And secondly, the very name is Algiebar, and not Algebra: as by the Arabian Avicenna may be proved, who has these precise words, translated into Latin by Andreas Alpaga (most perfect in the Arabic tongue): The Science of working Algiebar and Almachabel, that is, the Science of finding an unknown number, by adding a number, and by division and equation. Here have you the name, and also the principal parts of the rule, touched. To name it The Rule or Art of Equation signifies the middle part and the state of the rule. This Rule has its peculiar characters:

(5.)

and the principal parts of arithmetic pertaining to it do differ from the other arithmetic operations. This arithmetic has numbers simple, compound, mixed, and fractions accordingly. This Rule and arithmetic of Algiebar is so profound, so general, and so (in a manner) contains the whole power of the practical application of numbers, that man’s wit can deal with nothing more profitable about numbers, nor match with a thing more fit for the divine force of the Soul (in human studies, affairs, or exercises) to be tried in. Perhaps you looked for (long before now) to have had some particular proof or evident testimony of the use, profit, and commodity of vulgar arithmetic in the common life and

trade of men. To that, then, I will now frame myself; but in this I have great care, for fear that the length of sundry proofs might make you think that either I doubted your zealous mind in the school of virtue, or mistrusted your able wits to guess much more by themselves. A proof, then, four, five, or six such will I bring as any reasonable man may by that be persuaded to love and honor, yes even to learn and exercise, the excellent Science of Arithmetic.

And first: who, nearer at hand, can be a better witness of the fruit received by arithmetic than all kinds of merchants? Though not all alike either need it or use it. How could they forgo the use and help of the rule called the Golden Rule — simple and compound, both forward and backward? How might they miss arithmetic help in the Rules of Fellowship, either without time or with time, and between the merchant and his factor? The Rules of Bartering, in wares only, or part in wares and part in money, would they gladly do without? Our merchant venturers and travelers overseas, how could they order their dealings justly and without loss, unless certain and general rules for exchange of money and re-exchange were devised for their use? The Rule of Alligation, in how sundry cases does it conclude for them such precise truths as neither by natural wit nor by other experience were they able otherwise to know? And (with the merchant then to make an end) how ample and wonderful is the Rule of False Positions, especially as it is now enlarged by two excellent mathematicians of my familiar acquaintance in their lifetime — I mean Gemma Frisius and Simon Jacob? Who can either in brief conclude the general and capital rules? Or who can imagine the myriads of sundry cases and particular examples, in act and earnest, continually wrought, tried, and concluded by the forenamed rules alone? How many other arithmetic practices are commonly in merchants' hands and knowledge, they themselves can at large testify.

The mintmaster and goldsmith, in their mixture of metals, either of diverse kinds or diverse values, how are they, or may they, exactly be directed and wonderfully pleased, if arithmetic be their guide? And the honorable physicians will gladly confess themselves much beholden to the Science of Arithmetic, and that in sundry ways; but chiefly in their Art of Graduation and compound medicines. And though Galen, Averroes, Arnoldus, and Lullus have published

their positions, as well in the quantities of the Degrees above temperament as in the rules concluding the new Form resulting, yet a more precise, convenient, and easy Method is extant, invented by a countryman of ours

(Invented above 200 years ago.) And since I am uncertain who has the same, or when that little Latin treatise (as the author wrote it) shall come to be printed — both to declare the desire I have to pleasure my country as far as I may, and also for very good proof of the use of numbers in this most subtle and fruitful philosophical conclusion — I intend in the meanwhile, most briefly and with my further help, to communicate its pith to you.

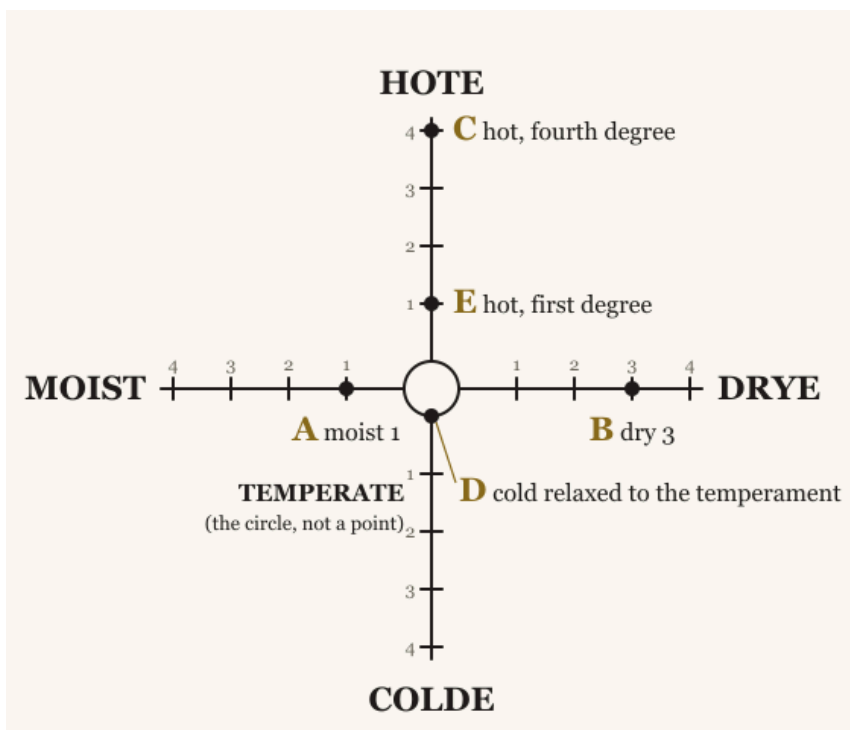
First draw a circle whose diameter is an inch. Divide the circumference into four equal parts. From the center, by those 4 sections, extend 4 straight lines: each $4\frac{1}{2}$ inches long, or as many as you please, above 4, outside the circumference of the circle; so that they shall be 4 inches long (at the least) outside the circle. Make clear and evident marks at the end of every inch. If you wish, you may subdivide the inches again into 10 or 12 smaller equal parts. At the ends of the lines write the names of the 4 principal elemental qualities: Hot and Cold, one opposite the other; and likewise Moist and Dry, one opposite the other. And in the circle write Temperate. Which temperature has a good latitude: as appears by the complexion of man. And therefore we have allowed to it the said circle, and not a mathematical or physical point.

(* Take some of Lullus's counsel in his book On the Fifth Essence.)

Now, when you have two miscible things whose degrees are * truly known: of necessity either they are of one quantity and weight, or of diverse. If they are of one quantity and weight: whether their forms are contrary qualities, or of one kind (but of diverse intentions and degrees), or a temperate and a contrary, the form resulting from their mixture is in the middle between the degrees of the forms mixed. As for example, let A be moist in the first degree, and B dry in the third degree. Add 1 and 3, that makes 4; the half or middle of 4 is 2. This 2 is the middle, equally distant from A and B

(* Note.)

(for the * temperate quality is counted as none, and for it you must put a zero, if at any time it is in a mixture).



The four elemental qualities and their degrees

Counting then from B, 2 degrees toward A, you find it to be dry in the first degree; so is the form resulting from the mixture of A and B in our example. I will give you another example. Suppose you have two things, as C and D, and of C the heat to be in the 4th degree, and of D the cold to be relaxed even down to the temperament. Now for C you take 4, and for D you take a zero, which, added to 4, yields only 4. The middle, or half, of which is 2. Therefore the form resulting from C and D is hot in the second degree; for 2 degrees counted from C toward D end exactly in the second degree of heat. Of the third manner I will give also an example; which let be this:

I have a liquid medicine whose quality of heat is exalted in the 4th degree, as was C in the example foregoing; and another liquid medicine I have whose quality is heat in the first degree. Of each of these I mixed a like quantity.

Subtract here the less from the more, and divide the residue into two equal parts; of which the one part, either added to the less or subtracted from the higher degree, produces the degree of the form resulting from this mixture of C and E. As, if from 4 you subtract 1, there remains 3; the half of 3 is $1\frac{1}{2}$: add to 1 this $1\frac{1}{2}$, you have $2\frac{1}{2}$; or subtract from 4 this $1\frac{1}{2}$, you have likewise $2\frac{1}{2}$ remaining. Which declares the form resulting to be heat in the middle of the third degree.

(The Second Rule.)

“But if the quantities of two commixed things are diverse, and the intensities (of their miscible forms) are in diverse degrees and heights (whether those forms be of one kind, or of contrary kinds, or of a temperate and a contrary), what proportion the lesser quantity has to the greater, the same shall be between the difference which is from the degree of the form resulting to the degree of the greater quantity of the miscible thing, and the difference which is from the same degree of the form resulting to the degree of the lesser quantity. As for example. Let two pounds of liquor be given, hot in the 4th degree; and one pound of liquor be given, hot in the third degree.” I would gladly know the form resulting in the mixture of these two liquors. Set down your numbers in order, thus.

£. 2. Hot. 4.

£. 1. Hot. 3.

Arithmetic, 2



By the rule of algebra I have devised a very easy, brief, and general way of working in this case. Let us first suppose that the middle Form resulting is x , as that Rule teaches. And because, by our Rule here given, as the weight of 1 is to 2, so is the difference between 4 (the degree of the greater quantity) and x , to the difference between x and 3 (the degree of the thing in lesser quantity; and all the while, x always lies in a certain middle between the two heights or degrees). For the first difference I set $4 - x$, and for the second I set $x - 3$. And now again I say, as 1 is to 2, so is $4 - x$ to $x - 3$. Therefore, of these four proportional numbers, the first and the fourth multiplied, one by the other, make as much as the second and the third multiplied, the one by the other. Let these multiplications be made accordingly. And of the first and the fourth we have $x - 3$, and of the second and the third, $8 - 2x$. Therefore our equation lies between $x - 3$ and $8 - 2x$. Which may be reduced according to the art of algebra: for here, adding 3 to each part gives the equation thus: $x = 11 - 2x$. And again, contracting or reducing it: add $2x$ to each part; then you have $3x$ equal to 11, thus represented: $3x = 11$. Therefore, dividing 11 by 3, the quotient is $3\frac{2}{3}$: the value of our x , the Coss, or Thing, first supposed. And that is the height, or intension, of the Form resulting: which is heat in two thirds of the fourth degree. And here I set out the work, in conclusion, thus. The proof of this is easy: subtracting 3 from $3\frac{2}{3}$ leaves $\frac{2}{3}$. Subtract the same height of the Form resulting, which is $3\frac{2}{3}$, from 4, and $\frac{1}{3}$ remains: you see that $\frac{2}{3}$ is double $\frac{1}{3}$, as 2 pounds is double 1

pound. So it should be, by the Rule here given. Note: as you added 3 to each part of the equation, so if you had first added $2x$ to each part, it would stand: $3x - 3 = 8$. And now, adding 3 to each part, you have, as before, $3x = 11$.

2 pounds, hot in the 4th degree: $\frac{1}{3}$; the Form resulting: $3\frac{2}{3}$; 1 pound, hot in the 3rd degree: $\frac{2}{3}$.

And though I speak here only of two miscible things, when most commonly three, four, five, or six, etc., are to be mixed (and reduced into one compound, with the Form resulting from the same to serve the turn), yet these Rules are sufficient, duly repeated and iterated.

In proceeding first with any two, and then with the Form Resulting and another, and so forth: for the last work concludes the Form Resulting of them all. I need say nothing of what the Mixture here supposed is. Common Philosophy has defined it, saying: Mixture is the union of miscible things that have been altered and joined through their smallest parts. Every word in the definition is of great importance. I need spend no time either to show how the other manner of distributing degrees agrees with these Rules. Neither need I declare here the further use belonging to the Cross of Graduation, before described, to those who are capable of what I have already said. Nor need I specify with examples the manifold varieties to be ordered by the two general Rules aforesaid. The witty and studious have enough here; and those who are not able to attain to this without lively teaching, and in more detail, would need a fuller discussion than is fitting to be dealt with in this place. And others, perhaps, with a proud disdain will scorn this little, and would be unthankful for much more. I therefore conclude, and wish such as have modest and earnest philosophical minds to praise God highly for this, and to wonder that the profoundest and subtlest point concerning the mixture of Forms and natural Qualities is so matched and married with the simplest, easiest, and shortest way of the noble Rule of Algebra. Who can remain, therefore, unpersuaded to love, allow, and honor the excellent science of arithmetic? For here you may perceive that the little finger of arithmetic has more might and contrivance than a hundred thousand men of the middle sort are able to perform, or truly to conclude, without the help of it.

Now we shall be further certified, by the wise and valiant Captain, what help he has from the Rules of arithmetic in one of the arts belonging to him, and which the Greeks name:

Τακτικῇ.

Τακτικῇ. “That is, the skill of ordering soldiers in battle array in the best manner for all purposes.” This art depends so much upon the use of numbers and the mathematics, that Aelianus, the best writer of it, in his work to the Emperor Hadrian, by his perfection in the mathematics, being greater than others before him had, thinks his book surpasses all other excellent works written of that art up to his days. For of it have written Aeneas, Cyneas of Thessaly, Pyrrhus Epirota and his son Alexander, Clearchus, Pausanias, Euangelus, Polybius, familiar friend to Scipio, Eupolemus, Iphicrates, Posidonius, and very many other worthy captains, philosophers, and princes of immortal fame and memory: whose fairest flower in their garland (in this feat) was arithmetic, and a little perception of geometric figures. But in many other cases arithmetic stands the captain in great stead. As in proportioning the victuals for the army, whether it is remaining at a stand, or is suddenly to be increased with a certain number of soldiers, and for a certain time. Or by good art to diminish his company, so that the victuals serve the remainder longer, and for a certain determined time, if need so requires. And so, in his sundry other accounts, reckonings, measurings, and proportionings, the wise, expert, and circumspect captain will affirm the science of arithmetic to be one of his chief counselors, directors, and aiders. Which thing (by good means) was evident to the noble, the courageous, the loyal, and courteous

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John, late Earl of Warwick. Who was a young gentleman thoroughly known to very few. Although his vigorous valor, force, and skill in chivalrous feats and exercises, his humility, and friendliness to all men, were things openly perceived by the world. But what roots virtue had otherwise fastened in his breast, what rules of godly and honorable life he had framed for himself, what notable vices (in some then living) he took great care to avoid, what manly virtues in other noblemen (flourishing before his eyes) he unceasingly aspired after, what prow-

ess he purposed and meant to achieve, with what feats and arts he began to furnish and provide himself for the better service of his King and country, both in peace and war: these, I say, his heroic meditations, forecastings, and determinations, no two, I think, besides myself, can report so perfectly and truly. And therefore, in conscience, I count it my part, for the honor, advancement, and procuring of virtue, thus briefly to have put his Name in the Register of Immortal Fame.

To our purpose. This John, by one of his acts (besides many others, both in England and France, noted by me in him) disclosed his hearty love of virtuous sciences, and his noble intent to excel in martial prowess: when, with humble request and constant soliciting, he obtained the best rules (whether used in times past by Greeks or Romans, or in our own time, with new stratagems devised in this) for ordering all companies, sums, and numbers of men, many or few, appointed with one kind of weapon or more, with artillery or without, on horseback or on foot, to give or take the onset, to seem many when few, and few when many; to march in battle or on a journey, and to many such feats belonging to a pitched field, skirmish, or ambush:

This noble Earl died in the year 1554, scarce 24 years of age, having no issue by his wife, daughter to the Duke of Somerset.

And all these lifelike designs (most carefully) to be drawn on vellum parchment, with notes and peculiar marks as the art requires; and all these rules and arithmetic descriptions enclosed in a rich case of gold, he used to wear about his neck, as his most precious jewel and most trustworthy counselor. Thus was arithmetic enshrined in gold by him; of the fruit of Numbers he had good hope. Now, therefore, innumerable Numbers shall find his shrine in the praise of Numbers.

What need have I (for further proof to you) of the schoolmasters of Justice, to require testimony, how needful, how fruitful, how skillful a thing arithmetic is? I mean lawyers of all sorts. Undoubtedly the civilians can wonderfully declare how neither the ancient Roman laws, without good knowledge of the art of numbers, can be perceived, nor (Justice in infinite cases) without due proportion, narrowly considered, be executed. How justly, and with great knowledge

of the art, did Papinian institute a law of partition and allowance between man and wife after a divorce? But how Accursius, Baldus, Bartolus, Jason, Alexander, and finally Alciatus (otherwise notably well learned) jumble, guess, and err from the equity, art, and intent of the lawmaker, arithmetic can detect and convict, and clearly make the truth shine. Good Bartolus, tired in examining and proportioning the matter, and much encumbered with Accursius's Gloss, burst out and said: In the whole book there is no gloss harder than this, whose account or reckoning neither the scholars nor the doctors understand. etc. What can they say of Julianus's law beginning "If it was thus written," etc., of the testator's will justly carried out between the wife, son, and daughter? How can they perceive the equity of Africanius's arithmetic reckoning, where he treats of the Falcidian Law? How can they clear him from his reprovers and those who maintain them, such as Joannes, Accursius Hypolitus, and Alciatus? How justly and skillfully was Africanius's reckoning made, proportioning to the sums bequeathed the contributions of each part? Namely, for the hundred presently received, $17\frac{1}{7}$; and for the hundred received after ten months, $12\frac{6}{7}$: which make the 30 which were to be contributed by the legatees to the heir. For what proportion 100 has to 75, the same has $17\frac{1}{7}$ to $12\frac{6}{7}$: which is Sesquitertia, that is, as 4 to 3, which make 7. Wonderfully many places in the civil law require an expert arithmetician to understand the deep judgment and just determination of the ancient Roman lawmakers. But much more expert ought he to be who would be able to decide with equity the infinite variety of cases which do or may happen under every one of those civil laws and ordinances. By this you may now easily conjecture that in the canon law, and in the laws of the realm (which with us bear the chief authority), justice and equity might be greatly preferred, and skillfully executed, through due skill of arithmetic and the proportions belonging to it. The worthy philosophers and prudent lawmakers (who have written many books On the Commonwealth, how the best state of commonwealths might be procured and maintained) have very well determined of justice (which not only is the base and foundation of commonwealths, but also the total perfection of all our works, words, and thoughts), defining it,

Justice.

“to be that virtue by which, to everyone, is rendered that to him appertains.” God demands this at our hands: to be honored as God, to be loved as a father, to be feared as Lord and master. Our neighbor’s proportion is also prescribed by the Almighty lawmaker: which is, to do to others as we would have them do to us. These proportions are necessary in justice, commendable in duty, and are the life, strength, support, and flourishing of commonwealths. Aristotle in his *Ethics* (to fetch the seed of justice and the light of direction, for using and executing the same) had to fly to the perfection and power of numbers: for arithmetic and geometric proportions. Plato, in his book called *Epinomis* (which book is the treasury of all his doctrine), where his purpose is to seek a science which, when a man had it perfectly, he might seem and so be, in deed, wise. He, briefly discoursing of other sciences, finds them unable to bring it to pass; but of the science of numbers he says: That science, truly, which has taught mankind number, shall be able to bring it to pass. And I think that a certain God, rather than fortune, has given us this gift for our bliss. For why should we not judge him who is the Author of all good things to be also the cause of the greatest good thing, namely, Wisdom? There at length he proves Wisdom to be attained by good skill of numbers. With which great testimony, and the manifold proofs and reasons before expressed, you may be sufficiently and fully persuaded of the perfect science of arithmetic, to make this account: that

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of all sciences, next to theology, it is most divine, most pure, most ample and general, most profound, most subtle, most commodious and most necessary. Whose next sister is the absolute science of magnitudes: of which (by the direction and aid of him whose Magnitude is Infinite, and of us Incomprehensible) I now intend so to write, that both with the multitude and also with the magnitude of marvelous and fruitful truths, you (my friends and countrymen) may be stirred up and awakened to see what certain arts and sciences (to our unspeakable benefit) our heavenly father has prepared and revealed for us, by sundry philosophers and mathematicians.

Arithmetic, 3



Both Number and Magnitude have a certain original seed (as it were) of incredible property, which man is never able fully to declare. Of Number, a Unit, and of Magnitude, a Point, seem to be much alike as original causes: but the difference, nevertheless, is great. We defined a Unit to be a mathematically indivisible thing; a Point, likewise, we said to be a mathematically indivisible thing. And further, that a Point may have a certain determined situation: that is, that we may assign and prescribe a Point to be here, there, or over there, etc. In this, behold, our Unit is free, and can abide no bondage, nor be tied to any place or seat, divisible or indivisible. Again, because a Point may have a situation limited to it, a certain motion (to a place and from a place) is incident and belonging to a Point. But a Unit cannot be imagined to have any motion. A Point, by its motion, produces mathematically a line (as we said before), which is the first kind of magnitudes and most simple: a Unit cannot produce any number. A Line, though it is produced by a Point moved, yet does not consist of points: Number, though it is not produced by a Unit, yet does consist of units as a material cause. But formally,
Number.

Number is the union and unity of units. Which uniting and knitting is the workmanship of our mind, which, from distinct and discrete units, makes a Number, by a uniformity resulting from a certain multitude of units. And so every number may have its least part given, namely a unit: but of a Magnitude

(no, not even of a Line) can the least part be given, because an infinite division of it may be conceived. All Magnitude is either a Line, a Plane, or a Solid. Which Line, Plane, or Solid can be perceived by no sense, nor exactly represented by hand in any way, nor produced by Nature: but as Number (by degrees) came to our perception, so by visible forms we are helped to imagine what our mathematical Line is, what our Point is. So precise are our magnitudes that one Line is no broader than another, for they have no breadth; nor our planes any thickness; nor yet our bodies any weight, be they ever so large of dimension. Our bodies we can have smaller than either Art or Nature can produce any, and greater also than all the world can contain. Our least magnitudes can be divided into as many parts as the greatest. As a Line one inch long (with us) may be divided into as many parts as may the diameter of the whole world from East to West, or extended any way: what privileges, above all manual Art and the might of Nature, have our two mathematical sciences, to exhibit and to deal with things of such power, liberty, simplicity, purity, and perfection? And in them to proceed so certainly, so orderly, so precisely: as excellent is that mechanical workman judged who can most nearly approach the representing of works mathematically demonstrated?

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And our two sciences, remaining pure and absolute in their proper terms and in their own matter, to have and allow only such demonstrations as are plain, certain, universal, and of an eternal truth?

Geometry



This Science of Magnitude, its properties, conditions, and appurtenances, has commonly been called geometry by all philosophers, now and from the beginning. But, truly, with a name too base and scant for a science of such dignity and amplitude. And perhaps that name, by common and secret consent of all wise men, has hitherto been allowed to remain, so that it might carry with it a perpetual memory of the first and most notable benefit shown to common people by that science: which was, when boundaries and meres of land and ground were lost and confounded (as in Egypt, yearly, with the overflowing of the Nile, the greatest and longest river in the world), or ground bequeathed was to be assigned, or ground sold was to be laid out, or (when disorder prevailed) commons were distributed into severalties. For where, upon these and such like occasions, some by ignorance, some by negligence, some by fraud, and some by violence, did wrongfully limit, measure, encroach upon, or claim (by pretense of just content and measure) those lands and grounds: great loss, disquiet, murder, and war did full often ensue: till, by God's mercy and men's industry, the perfect Science of Lines, Planes, and Solids (like a divine justiciar) gave to every man his own. The people then, pleased by this art and greatly relieved in the just measuring of their lands, and other philosophers writing rules for land measuring, between them both thus confirmed the name Geometria, that is (according to the very etymology of the word) land measuring. In it, the people knew no further of the use of magnitudes but in planes; and the philosophers

had no fit hearers or scholars among them, to disclose further to than of flat, plain geometry. And though these philosophers knew of further use, and best understood the etymology of the word, yet this name Geometria was by them applied generally to all sorts of magnitudes: except at other times by Plato and Pythagoras, when they would precisely declare their own doctrine. Then it was Plato, Republic 7.

Geometria, with them, the study concerned with the plane. But well you may perceive by Euclid's Elements that our Science is more ample than to measure planes, and nothing less is taught in it (on purpose) than how to measure land. Another name, therefore, must be had for our mathematical Science of Magnitudes: which regards neither clod nor turf, neither hill nor dale, neither earth nor heaven, but is absolute Megethologia: not creeping on ground and dazzling the eye with pole, perch, rod, or line, but "lifting the heart above the heavens by invisible lines, and

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immortal beams meeting with the reflections of the incomprehensible light, and so procuring joy and unspeakable perfection." Of which true use of our Megethica, or Megethologia, Divine Plato seemed to have good taste and judgment, and (by the name of geometry) so noted it, and warned his scholars of it, as may clearly be seen in his seventh Dialogue of the Commonwealth. Where it is thus, rightly translated: Truly, says Plato, whoever has (even very little) tasted of geometry will not deny us this: that this Science is of another condition, quite contrary to that which they who are exercised in it speak of it. And there it follows, of our geometry: It must be confessed, says Plato, that [geometry] is learned for the knowing of that which is ever, and not of that which in time both is bred and is brought to an end. etc. Geometry is the knowledge of that which is everlasting. It will lift up, therefore (O Gentle Sir), our mind to the Truth, and by that means it will prepare the Thought to the philosophical love of wisdom: that we may turn or convert toward heavenly things [both mind and thought] which now, otherwise than becomes us, we cast down on base or inferior things. etc. Chiefly, therefore, commandment must be given that such as inhabit this most honorable City by no means despise geometry. For even

those things [done by it] which in manner seem to be beside the purpose of geometry are of no small importance. etc. And besides the manifold uses of geometry in matters pertaining to war, he adds more, of second unpurposed fruit and commodity arising from geometry, saying: But also we know that for the more easy learning of all arts it much imports whether one has any knowledge in geometry or no. etc. Let us therefore make an ordinance or decree that this Science shall be learned by young men in the second place. This was Divine Plato's judgment, both of the purposed, chief, and perfect use of geometry, and of his second, dependent, derivative commodities. And for us, Christian men, a thousand thousand more occasions are, to have need of the help of

I. D. In this, I would gladly shake off the earthly name of geometry.

metaphysical contemplations, by which to train our imaginations and minds little by little to forsake and abandon the gross and corruptible objects of our outward senses, and to apprehend, by sure demonstrative doctrine, Mathematical Things. And by them, readily to be helped and conducted to conceive, discourse, and conclude of things Intellectual, Spiritual, eternal, and such as concern our everlasting Bliss: which otherwise (without special privilege of Illumination or Revelation from heaven) no mortal man's wit (naturally) is able to reach or to compass. And truly, by my small Talent (from above) I am able to prove and testify that the literal Text and order of our divine Law, Oracles, and Mysteries require more skill in Numbers and Magnitudes than (commonly) the expositors have uttered: but rather only (at most) so warned, and showed their own want therein. (To name any is needless; and to note the places is here no place; but if I am duly asked, my answer is ready.) And without the literal, grammatical, mathematical, or natural truths of such places, perceived by good and certain Art, no spiritual sense (proper to those places by Absolute Theology) will depend thereon.

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"No man, therefore, can doubt but toward the attaining of knowledge incomparable, and Heavenly Wisdom: Mathematical Speculations, both of Numbers and Magnitudes: are means, aids, and guides: ready, certain, and necessary." From from now on, in this my Preface, will I frame my talk to Plato's fugitive

Scholars; or rather to such as well can (and also will) use their outward senses, to the glory of God, the benefit of their Country, and their own secret contentment, or honest advancement, on this earthly Scaffold. To them I will orderly recite, describe, and declare a great Number of arts, from our two mathematical fountains, derived into the fields of Nature. By which such Seeds and Roots as lie deep hidden in the ground of Nature are refreshed, quickened, and provoked to grow, shoot up, flower, and give fruit, infinite and incredible. And these arts shall be such as depend upon the properties of Magnitudes more than upon Number. And by good reason we may call them arts, and derived mathematical arts: for (at this time) I define

An art.

An art to be a methodical complete doctrine, having abundance of sufficient and peculiar matter to deal with, by the allowance of the metaphysical philosopher: the knowledge of which is necessary to the human estate. And that I count,

Derived mathematical art.

A derived mathematical art, which by a mathematical demonstrative method, in Numbers or Magnitudes, orders and confirms its doctrine, as much and as perfectly as the subject matter will admit. And for that, I intend to use the name and property of a

A mechanician.

mechanician, otherwise than (hitherto) it has been used, I think it good (for distinction's sake) to give you also a brief description of what I mean by that. A mechanician, or a mechanical workman, is he whose skill is, without knowledge of mathematical demonstration, perfectly to work and finish any sensible work, by the mathematician, principal or derivative, demonstrated or demonstrable. Full well I know that he who invents or makes these demonstrations is generally called a speculative mechanician, which differs nothing from a mechanical mathematician. So, in respect of diverse actions, one man may have the names of sundry arts: as sometimes a logician, sometimes (in the same matter otherwise handled) a rhetorician. Of these trifles I make (as now, in respect

of my Preface) small account: to file them for the fine handling of subtle curious disputers. In other places they may command me to give good reason: and yet here I will not be unreasonable.

(1.)

First, then, from the purity, absoluteness, and immateriality of Principal Geometry is derived that kind of geometry which vulgarly is counted geometry: and is the Art of measuring sensible magnitudes, their just quantities and contents.

Common Geometry



This teaches measuring either at hand, with the practitioner standing by the thing measured: and so, by duly applying the compass, rule, square, yard, ell, perch, pole, line, gaging rod (or the like instrument) to the length, plane, or solid being measured,

(1.)

To be certified of the length, perimeter, or linear distance: and this is called Mecometry. Or

(2.)

To be certified of the content of any plane surface: whether it be surveyed ground, a measured board or pane of glass, or the like thing: which measuring is named Embadometry.

(3.)

Or else to understand the solidity and capacity of any bodily thing: such as timber and stone, or the capacity of pits, ponds, wells, vessels small and great of all kinds. Where the measuring of wine, oil, beer, or ale vessels, etc., commonly has a peculiar name: and is called gaging. And the general name of these solid measures is stereometry.

(2.)

Or else this common geometry also undertakes to teach the practitioner how to measure things while standing at a good distance from the thing measured: and by that to understand either

(1.)

How far a thing seen (on land or water) is from the measurer: and this may be called Apomecometry:

(2.)

Or, how high or deep, above or below the level of the measurer's standing, anything is that is seen on land or water, called Hypsometry.

(3.)

Or, it informs the measurer how broad anything is that lies in the measurer's view, provided it is situated on land or water: and may be called Platometry. Though I am in the habit of stipulating that the thing measured be situated on land or water:

Yet know for certain that the various heights of clouds, comets, and the moon may by these means have their distances from the earth; and that of comets and the moon, the solidity as well as the distances may be measured. But because neither these things are taught in the common way, nor are so ready for the ordinary practitioner to carry out, I prefer to let such measures be reckoned as incidental to some of our other arts, which deal more purposely with things on high than this common land-measuring geometry does: as in perspective and astronomy, etc.

From these feats, further applied, has sprung the feat of geodesy, or land surveying: to measure and survey land, woods, and waters more skillfully from afar. More skillfully, I say: but God knows, up to now in these realms of England and Ireland (whether through ignorance or fraud, I cannot tell in every particular)

How great a wrong and injury has been done in my time by untrue measuring and surveying of land or woods in every way. And of this I am sure: that the value of the difference between the truth and such surveys would have been able to fund forever, in each of our two universities, an excellent mathematics reader: each allowed yearly a hundred marks of lawful money of this realm; which indeed would seem to be required here (though provided for in other

ways), just as the famous University of Paris has two mathematics readers, each with two hundred French crowns yearly, from the French king's magnificent liberality alone.

Now, returning again to our purpose: moreover, from that earlier geometrical knowledge have grown the skills of geography, chorography, hydrography, and Stratarithmetrie.

Geography



“Geography teaches the ways by which, in various forms (as spherical, planar, or other), the situation of cities, towns, villages, forts, castles, mountains, woods, harbors, rivers, creeks, and such other things upon the surface of the earthly globe (either in the whole, or in some principal member and portion of it contained) may be described and drawn in proportions analogous to nature and truth, and represented most aptly to our view.” Of this art how great pleasure and how manifold the commodities that come to us daily and hourly, most men perceive. While some, to beautify their halls, parlors, chambers, galleries, studies, or libraries; others, for things past, as battles fought, earthquakes, heavenly fires, and such occurrences mentioned in histories: by that livelike, as it were, to view the place, the neighboring region, the distance from us, and such other circumstances. Others, presently to view the wide dominion of the Turk, the vast empire of the Muscovite, and the little morsel of ground where Christendom (by profession) is certainly known. Little, I say, in respect of the rest, etc. Some, either to direct their own journeys into far countries, or to learn of other men’s travels. To conclude, some for one purpose and some for another like, love, get, and use maps, charts, and geographic globes. Of whose use to speak adequately would require a book of its own.

Chorography



Chorography seems to be an underling and a twig of geography: and yet nevertheless is manifold in practice and very ample in use. “This teaches, analogically, to describe a small portion or circuit of ground with its contents: without regard to what commensuration it has to the whole, or any parcel contained outside it. But in the territory or parcel of ground it takes in hand to describe, it leaves out nothing notable or odd standing above ground and visible. And sometimes it gives some peculiar mark or warning of things underground: as of metal mines, coal pits, stone quarries, etc.” Thus a duchy, a shire, a lordship, or less may be described distinctly. But it is marvelous pleasant and profitable to set before our eye, and in commensuration, the plan of a city, town, fort, or palace in true symmetry: without approaching any of them, and out of gunshot, etc. By that the architect can furnish himself with a store of whatever patterns he likes: to his great instruction, even in those things that are outwardly proportioned: either simply in themselves, or respectively to the hills, rivers, harbors, and woods adjoining. Some also call this particular description of places topography.

Hydrography



“Hydrography gives us, on globe or in plane, the perfect analogical description of the ocean sea coasts throughout the whole world, or in its chief and principal parts:” with the islands and chief particular places of danger contained within the bounds and sea coasts described: such as quicksands, banks, pits, rocks, races, counter-tides, whirlpools, etc. This deals chiefly with the element of water: just as geography did principally take in hand the description of the element of earth (with its appurtenances). And besides this, hydrography requires a particular register of certain landmarks (where marks can be had) well able to be spied from the sea: at what point of the sea compass they appear, and what apparent form, situation, and bigness they have in respect of any dangerous place in the sea, or near to it, assigned. And on all coasts, what moon makes full sea, and in what way the tides and ebbs come and go, the hydrographer ought to record. The soundings likewise, and the channel ways: their number and depths ordinarily at ebb and flood, the hydrographer ought by observation and diligent measuring to have certainly known. And many other points belong to perfect hydrography, and to making sailing directions: of which I need not speak here: such as truly describing, in any place, on globe or plane, the 32 points of the compass (of which scarcely four in England have right knowledge, because the lines of it are neither straight lines nor circles). Of making due projection of a sphere in plane. Of the variation of the compass from true north. And such like matters (all of great importance) I leave to speak of here: because

I may seem already to have widened the bounds and duty of a hydrographer far more than any man (to this day) has noted or prescribed. Yet I am well able to prove that all these things pertain, and are also proper, to the hydrographer. The chief use and end of this art is the art of navigation: but it has other diverse uses: even to be enjoyed by those who never lack sight of land.

Stratarithmetrie



Stratarithmetrie is the skill (appertaining to war) by which a man can set in a figure, analogous to any geometric figure appointed, any certain number or sum of men: capable of such a figure (by reason of the usual spaces allowed between soldiers, and because no fractions can be made of men. Yet nevertheless he can order the given sum of men into the greatest such figure that can be ordered of them) and certify the surplus (if any) and the next certain sum which, with the surplus, will admit a figure exactly proportional to the assigned figure. By which skill also, of any army or company of men (the figure and sides of whose orderly standing or array is known), he is able to express the exact number of men contained within that figure: or (in order) able to be contained. Note.

And this figure and its sides he is able to know: either being by and at hand, or far off. Thus far stretches the description and property of Stratarithmetrie: sufficient for this time and place.

The difference between Stratarithmetrie and Tacticie.

“It differs from the tactical feat of drawing up battle lines, because there it takes wisdom and foresight to what purpose he orders the men; and skillful ability also, for any occasion or purpose, to devise and use the fittest and most necessary order, array, and figure of his company and sum of men.” By figure I mean such as a perfect square, triangle, circle, oval, long square (which the Greeks call Eteromekes), rhomb, rhomboid, crescent, ring, serpentine, and such other geo-

metric figures: which in wars have been and are to be used for convenience, necessity, and advantage, etc. And he ought to have no small skill who can give a true report, or near the truth, of the numbers and sums of footmen or horsemen in the enemy's ordering. To make an estimate from afar, between close terms of more and less, is not a thing very common among those who would gladly do it.

I. D., friend, you will find it hard to carry out my description of this feat. But by chorography you can help yourself somewhat: where the figures known (in sides and angles) are not regular, and where a resolution into triangles can serve, etc. And yet you will find it strange to deal so generally with arithmetic figures, and that for battle array: their contents differ so much from like geometric figures.

Great policy may be used by the captains (at fitting times and in convenient places): as to use figures that make the greatest show of as many men as he has; and using the advantage of the three kinds of usual spaces (between footmen or horsemen) to take the largest; or when he would seem to have few, being many: and the contrary, in figure and space. The herald, pursuivant, sergeant royal, captain, or whoever is careful to come near the truth in this, besides the judgment of his expert eye, his skill of tactical ordering, and the help of his geometric instrument, ring or astronomical staff (conveniently framed for carrying and use), he may wonderfully help himself with perspective glasses. In which (I trust) our posterity will prove more skillful and expert, and for greater purposes, than in these days can almost be credited to be possible.

Thus I have lightly passed over the artificial feats depending chiefly upon common geometry, and commonly and generally reckoned under the name of geometry.

The Mixed Sciences and Perspective



But there are other (very many) methodical arts, which, declining from the purity, simplicity, and immateriality of our principal science of magnitudes, do nevertheless use the great aid, direction, and method of the said principal science, and have proper names, distinct both from the science of geometry (from which they are derived) and from one another: such as perspective, astronomy, music, Cosmography, astrology, statics, anthropography, Trochilike, Helicosophy, Pneumatithmie, Menadrie, Hypogeiodie, Hydragogie, horometry, painting, architecture, navigation, Thaumaturgike and Archemastrie. I think it necessary to give, in order, some particular description of these, and also to touch on some of their useful applications, and so to make this preface a little sweet, pleasant nosegay for you: to comfort your spirits when they are almost out of courage and in despair (through a brutish turn of mind), supposing that geometry has served only for the building of a house, or an ornate bridge, or the roof of Westminster Hall, or some clever little device or engine suited to a carpenter or a joiner, etc. That things are far otherwise than the world has commonly judged them, to this day, will be well proved both in word and in deed.

Among these arts, by good reason, perspective ought to be had before perfect knowledge of astronomical appearances can be attained. And because light holds the prerogative as the first of God's creatures, and the eye is the light of our body, whose sense is the mightiest and whose organ is the most skillful and geometric, we will begin, therefore, with perspective. Perspective is a mathematical art which demonstrates the manner and properties of all radiations—direct, broken, and reflected. This description, or statement, is brief, but it reaches as far as the world is wide. It concerns all creatures, all actions and passions brought about by the emanation of rays. By rays, or natural lines, I mean here not only those of light or color (though these give the eye appearance, witness, and proof on which to ground the art) but also the certain and determined active radial emanations of other forms, both substantial and accidental. Through this art (setting aside its highest points) we may use our eyes and the light with greater pleasure and more perfect judgment, both of things seen in light and of other things, which work and produce their effects by the same order of the radiation of lights. We should be ashamed to be ignorant of the cause of the many ways our eye is deceived and misled: as when the eye takes a round globe or sphere, far off, to be a flat plain circle, and likewise judges a flat square to be round; supposes parallel walls to draw together at a distance; and takes roof and floor, parallel to each other, the one to bend downward and the other to rise up, at a little distance from you. Again, of things moving at equal speed, to think the nearer moves faster and the farther much slower. And of two things, one of which moves incomparably faster than the other, to judge the slower to move very swiftly and the other to stand still—what an error is this of our eye! Of the rainbow: of its colors, the order of the colors, its size, its place and height, etc.—to know the demonstrative causes, is it not pleasant, is it not necessary? Of two or three suns appearing, of blazing stars, and such things, brought to pass by natural causes (and yet, nevertheless, significant of matters beyond): is it not useful for humankind to know the very true cause and the natural occasion? Is it not, rather, greatly beneath the dignity of human nature to be so misled and deceived by things at hand, before one's eyes: as with a peacock's tail and a dove's neck, or a whole oar held in water seeming broken?

Distant things seeming near, and near things seeming distant. Small things seeming great, and great things seeming small. One man seeming an army. Or a person being terribly afraid of his own shadow. Yes, to fear so much that, if you are alone near a certain mirror and make as if to thrust at it with a dagger or sword, you will suddenly be moved to draw back, as it were, by reason of an image,

A marvelous mirror.

appearing in the air between you and the mirror, with the same hand, sword or dagger, and with equal quickness, lunging at your very eye just as you do at the mirror. Strange this is to hear of, but more marvelous to see than these words of mine can express. And yet by optical demonstration the order and cause of it are made certain, even as its effect follows from it. Yes, this much more I dare undertake, toward satisfying the noble spirit that longs ardently for the wisdom of natural causes: to let him understand that in London he may with his own eyes have proof of what I have said here. A gentleman (who, for the good service he has done his country, is famous and honorable:

and for skill in the mathematical sciences and languages is the odd man of this land, etc.) Even he is able, and I am sure will very willingly, let the mirror and the proof be seen; and so I here request it of him: for the increase of wisdom among the honorable, for the stopping of malicious mouths, and for the repressing of the arrogance of the ignorant. You may easily guess what I mean. This art of perspective is of such excellence, and may be led to the establishing and carrying out of such things as no one would readily believe without actual proof perceived. I say nothing of natural philosophy, which, without perspective, cannot be fully understood nor perfectly attained. Nor of astronomy, which, without perspective, cannot well be grounded; nor of astrology, verified and confirmed on natural grounds. That part of it which deals with mirrors (the word mirror being, in this art, a general name for anything from which a ray bounces back) is called catoptrics, and has so many uses, both marvelous and profitable, that it would keep me too long to note the principal conclu-

sions already known in it; and also some things might perhaps lack due credit with you, and I would by that lose my labor, and you would slip into a light judgment,

Before you have learned sufficiently the power of nature and art.

Astronomy



Now, to proceed: Astronomy is a mathematical art which demonstrates the distances, magnitudes, and all natural motions, appearances, and affections proper to the planets and fixed stars: for any time past, present, or to come: with respect to a given horizon, or without respect to any horizon. By this art we are assured of the distance of the starry sky and of each planet from the center of the Earth, and of the size of any fixed star seen, or of any planet, in proportion to the Earth's size. Thus we are sure (by this art) that the solidity, bulk, and body of the Sun contain the quantity of the whole Earth and Sea a hundred and sixty-two times, less by one eighth part of the Earth. But the body of the whole earthly globe and Sea is bigger than the body of the Moon forty-three times, less by one eighth of the Moon. Therefore the Sun is bigger than the Moon 7000 times, less by $59 \frac{39}{64}$ —that is, precisely $6940 \frac{25}{64}$ times bigger than the Moon. And yet an unskillful person would judge them alike in size. Therefore, necessarily, one is much farther from us than the other. The Sun, when it is farthest from the Earth (which now, in our age, is when it is in the 8th degree of Cancer), is 1179 semidiameters of the Earth distant. And the Moon, when she is farthest from the Earth, is 68 semidiameters of the Earth and $\frac{1}{3}$. The nearest the Moon comes to the Earth is $52\frac{1}{4}$ semidiameters. The distance of the starry sky from us, in semidiameters of the Earth, is $20081\frac{1}{2}$ —

twenty thousand, eighty-one and a half. Subtract from this the Moon's nearest distance from the Earth, and there remain $20029\frac{1}{4}$ semidiameters of the Earth—twenty thousand twenty-nine and a quarter.

So thick is the heavenly palace in which the planets have all their exercise, and so marvelously do they perform the command and charge given to them by the omnipotent majesty of the King of kings. This is that which in Genesis is called Ha Rakia. Consider it well. The semidiameter of the Earth contains, in our common miles, $3436\frac{4}{11}$ —that is, three thousand four hundred thirty-six and four elevenths of one mile; so that the whole Earth and Sea round about is 21600, twenty-one thousand six hundred of our miles, allowing sixty miles for every degree of the greatest circle. Now if you weigh well within yourself but this little parcel of astronomical fruit, concerning the size and distances of the Sun, Moon, and starry sky, and the huge mass of Ha Rakia, will you not find your conscience moved, with the kingly prophet, to sing the confession of God's glory, and say, "The heavens declare the glory of God, and the firmament [Ha Rakia] shows forth the works of his hands." And so forth for those first five staves of that kingly psalm. Well, well, it is time for some to lay hold of wisdom and to judge things truly, and not so to expound the Holy Word all by allegories, as to neglect the wisdom, power, and goodness of God to be seen and learned in and through his creatures and creation. Through parables and analogies of their natures and properties, the course of Holy Scripture also declares to us very many mysteries. The whole frame of God's creatures (which is the whole world) is to us a bright mirror, from which, by reflection, beams and radiations bounce back to our knowledge and perception, representing the image of his infinite goodness, omnipotence, and wisdom. And we are by that taught and persuaded to glorify our Creator as God, and to be thankful for it. Could the heathens find these uses in these most pure, beautiful, and mighty corporeal creatures, and shall we—after the true Sun of righteousness has risen above the horizon of our temporal hemisphere, and has so abundantly streamed into our hearts the direct beams of his goodness, mercy, and grace, whose heat all creatures feel, spiritual and corporeal, visible and invisible? Shall we, I say, look upon the heaven, stars, and planets as an ox and a donkey do, no

further caring or inquiring what they are, why they were created, or how they carry out the purpose for which they were created? Seeing that all creatures were created for our sake, and that both we and they were created chiefly to glorify the Almighty Creator, and that by every means possible to us. Do not be ignorant (says Plato in *Epinomis*) that astronomy is a thing of excellent wisdom. Astronomy was from the beginning commended to us, and in a manner commanded by God himself, since he made the sun, moon, and stars to be for us signs, and knowledge of seasons, and for the distinction of days and years. Many words are not needed. But I wish every person would weigh this word, signs. And besides that, compare it also with the tenth chapter of *Jeremiah*. And though some think that there they have found a rod, yet modest reason will be an impartial judge of who ought to be beaten with it, in respect of our purpose. Leaving that, I ask you to understand this: that without great diligence of observation, examination, and calculation, their periods and courses (by which the distinction of seasons, years, and new moons might be precisely known) could not be exactly established. To accomplish this is the art which we have here defined as astronomy. By it we may have the distinct course of times, days, years, and ages: both for the consideration of sacred prophecies foretold and accomplished in due time, and for the holding of high mystical solemnities, and for all other human affairs, conditions, and covenants made at a fixed time between person and person, with many other great uses: in which, truly, there would be great uncertainty, confusion, untruth, and brutish barbarism without the wonderful diligence and skill of this art, continually learning and determining times and periods of time by the record of the heavenly book in which all times are written, to be read with an astronomical staff instead of a pointer.

Music



Music has its original cause in motion: therefore, after the motions most swift and most slow, which are in the firmament, performed by nature and under the astronomer's consideration, now I will speak of another kind of motion, producing sound, audible and countable by man. By music I mean, here, that science which the Greeks call Harmonice. I do not meddle with the controversy between the ancient Harmonists and Canonists. Music is a mathematical science which teaches, by sense and reason, perfectly to judge and order the diversities of sounds, high and low. Astronomy and music are sisters, says Plato. As, for astronomy, the eyes were made; so, for harmonious motion, the ears. But as astronomy has a more divine contemplation and benefit than mortal eye can perceive, so music is to be considered,

(1.)

that the mind may be preferred to the ear. And from audible sound we ought to rise to the examination: which numbers are harmonious and which are not. And why the one are, and the other are not. I could at length,

(2.)

in the heavenly motions and distances, describe a marvelous harmony, of Pythagoras' harp

(3.)

with eight strings. Also, something might be said of Mercury's two harps,

(4.)

each of four elemental strings. And very strange matter might be alleged of the harmony,

(5.)

appropriate to our spiritual part. As may appear in Ptolemy's third book, in the fourth and sixth chapters.

(6.)

And what is the cause of the fitting bond and friendly fellowship of the intellectual and mental part of us with our gross and corruptible body, but a certain mean and harmonious spirituality, participating in both and (in a manner) resulting in

(7.)

the tune of the human voice, and also

(8.)

the sound of instruments: what might be said of harmony, no common musician would readily believe.

I. D. Read in Aristotle's 8th book of Politics, the 5th, 6th, and 7th chapters. Where you will have further occasion to think about music than is commonly thought.

But of the sundry mixture (as I may call it) and concourse, the diverse comparison and application of these harmonies, as of three, four, five, or more: marvelous have the effects been; and yet the like may be found and produced, with some proportional consideration for our time and being, in respect of the state of the things then, in which and by which the wondrous effects were wrought. Democritus and Theophrastus affirmed that, by music, griefs and diseases of the mind and body might be cured, or inferred. And we find in record that Terpander, Arion, Ismenias, Orpheus, Amphion, David, Pythagoras, Empedocles, Asclepiades, and Timotheus, by harmonic consonance, have done and brought to pass things more than marvelous to hear of. Of them, then, making no further discourse here: I am sure that common music, commonly used, is found by musicians and hearers to be so beneficial and pleasant that if I were to say and argue only this much—that it ought to be used otherwise than it is—I should find more critics than I could find people privy to, or skilled in,

my meaning. In things, therefore, evident and better known than I can express, and so allowed and approved of (as I would wish some other things had the like luck), I will spare to enlarge my lines any further, but consequently follow my purpose.

Cosmography



Of cosmography, I have briefly set out here to give you some information. Cosmography is the whole and perfect description of the heavenly and also the elemental part of the world, and their corresponding application and necessary mutual comparison. This art requires astronomy, geography, hydrography, and music. Therefore it is no small art, nor so simple as it is (slightly) considered in common practice. This matches heaven and earth in one frame, and fittingly applies corresponding parts, so that the heavenly globe may (in practice) be duly described upon the geographic and hydrographic globe. And there, for us to consider an equinoctial circle, an ecliptic line, colures, poles, stars in their true longitudes, latitudes, declinations, and verticality; also climes and parallels: and by an annexed horizon, and the revolution of the earthly globe (as the heaven is carried about by the primum mobile, the outermost moving sphere, in 24 equal hours), to learn the risings and settings of stars (of Virgil in his Georgics: of Hesiod: of Hippocrates in his medical sphere, to Perdicca, king of the Macedonians: of Diocles, to King Antigonus, and of other famous philosophers prescribed), a thing necessary for the due tillage of the earth, for navigation, for the alteration of a man's body, being whole, sick, wounded, or bruised. By the revolution, also, or motion of the cosmographic globe, the rising and setting of the sun, the lengths of days and nights, and the hours and times

(both night and day) are known: with very many other pleasant and necessary uses: of which some are known, but the better remain, for such as know and use them:

(☞)

who, from a spark of true fire, can make a wonderful bonfire, by duly applying due matter.

Astrology



Of astrology, here I make an art, separate from astronomy: not by new device, but by good reason and authority: for astrology is a mathematical art which reasonably demonstrates the operations and effects of the natural beams of light and the secret influence of the stars and planets: in every element and elemental body: at all times, in any given horizon. This art is furnished with many other great arts and experiences: as with perfect perspective, astronomy, cosmography, natural philosophy of the 4 elements, the art of graduation, and some good understanding in music: and yet moreover with another great art, following hereafter, though I set this one before, for some considerations that move me. Sufficient (you see) is the stuff for making this rare and secret art; and hard enough to shape into a syllogistic conclusion. Yet both the manifold and continual travels of the most ancient and wise philosophers, for the attaining of this art, and, by examples of effects, to confirm it, have left us sufficient proof and witness; and we also may perceive daily that a man's body, and all other elemental bodies, are altered, disposed, ordered, pleased, and displeased by the influential working of the sun, the moon, and the other stars and planets. And therefore, says Aristotle, in the first of his Meteorological books, in the second chapter: This [elemental] world is of necessity almost adjoining the heavenly motions: that from there all its virtue or force may be governed. For that is to be thought the first cause of all: from which the beginning of motion is. And again, in the tenth chapter: We must therefore also take up the principles of

these things, and the causes of all alike. The principle, then, as that which moves, chief and first of all, is that circle in which the sun's motion is plainly seen, etc. And so forth. His Meteorological books are full of arguments and effective demonstrations of the virtue, operation, and power of the heavenly bodies in and upon the four elements, and other bodies composed of them (either perfectly or imperfectly). And in his second book, On Generation and Corruption, in the tenth chapter: Therefore the uppermost motion is not the cause of generation and corruption, but the motion of the zodiac: for that both is continual and is caused by two motions. And in his second book, and second chapter of his Physics. For man (says he) and the sun are the cause of man's generation. Authorities may be brought, very many: of 1000, 2000, yes and 3000 years of antiquity: from great philosophers, expert, wise, and godly men, for that conclusion: which daily and hourly we men may discern and perceive by sense and reason: all beasts feel, and simply show, by their actions and passions, outward and inward: all plants, herbs, trees, flowers, and fruits. And finally, the elements, and all things composed of the elements, give testimony (as Aristotle said) that their whole dispositions, virtues, and natural motions depend on the activity of the heavenly motions and influences. By which, besides the specific order and form due to every seed: and besides the nature proper to the individual matrix of the thing produced: what the heavenly impression shall be, the perfect and circumspect astrologer has to conclude. Not only (by apotelesms) τὸ ὅτι, but by natural and mathematical demonstration τὸ διότι. As to which, what sciences are requisite (without exception) I have partly warned here: and in my Propædeumes (besides other matter disclosed there) I have mathematically furnished up the whole method: which in this age of ours has not been so carefully handled by anyone I ever saw or heard of. I was,

Years 1548 and 1549, in Louvain.

(for 21 years ago) by certain earnest disputations of the learned Gerardus Mercator and Antonius Gogaua (and others), so provoked thereto: and (by my constant and invincible zeal to the truth) in observations of heavenly influences (to the minute of time), then so diligent: and chiefly by the supernatural influ-

ence from the star of Jacob, so directed: that any modest and sober student, carefully and diligently seeking the truth, will both find and confess in this the truth of these my words: and also become a reasonable reformer of three sorts of people, greatly erring from the truth about these influential operations.

Of these, the one is light believers, the other light despisers, and the third light practitioners. The first and most common sort think the heaven and stars answerable to any of their doubts or desires:

(1.)

which is not so: and indeed they overreach too much. The second sort think no influential virtue (from the heavenly bodies) bears any sway in generation

(2.)

and corruption, in this elemental world. And to the sun, moon, and stars (being so many, so pure, so bright, so wonderfully big, so far in distance, so manifold in their motions, so constant in their periods, &c.) they assign a slight, simple office or two, and so allow them (according to their capacities) as much virtue and influential power as they allow to the sign of the sun, moon, and seven stars, hung up (for signs) in London to distinguish houses, and such gross helps in our worldly affairs: and they do not understand (or will not understand) the other workings and virtues of the heavenly sun, moon, and stars: not so much as the mariner or husbandman: no, not so much as the elephant does, as the Cynocephalus, as the porcupine does; nor will they allow these perfect and incorruptible mighty bodies so much virtual radiation and force as they see in a little piece of magnet stone, which at a great distance shows its operation. And perhaps they think the sea and rivers (as the Thames) to be some living thing, and so to ebb and flow, run in and out, of themselves at their own fantasies. God help, God help. Surely these men come short: and either are too dull, or willfully blind, or perhaps too malicious. The third man is the common and vulgar astrologer, or practitioner: who, not being duly, skillfully, and perfectly

(3.)

furnished: yet either for vain glory or gain, or like a simple dolt and blind Bayard, errs both in matter and manner: to the discredit of the wary and modest astrologer, and to the robbing of those most noble corporeal creatures of their natural virtue: being most mighty, most beneficial to all elemental generation, corruption, and their appurtenances: and most harmonious in their monarchy: for which things, being known and modestly used, we might highly and continually glorify God, with the princely prophet, saying: The heavens declare the glory of God: who made the heavens in his wisdom: who made the sun to have dominion of the day: the moon and stars to have dominion of the night: by which day to day utters speech, and night to night declares knowledge. Praise him, all you stars, and light. Amen.

Statics, 1



In order, now follows, of Statike: to say something of what we mean by that name, and what benefit depends on such an art. Statike is a mathematical art, which demonstrates the causes of heaviness and lightness in all things, and of the motions and properties belonging to heaviness and lightness. And since by the bilanx, or balance (as the chief sensible instrument) experience of these demonstrations may be had, we call this art Statike: that is, the experiments of the balance. Oh, that men knew what profit, in every way, might grow from this art to the able examiner and diligent practitioner. “You alone know all things precisely (O God), who have made weight and balance your judgment; who have created all things in number, weight, and measure; who have weighed the mountains and hills in a balance, and poised in your hand both heaven and earth. We therefore, warned by the sacred word to consider your creatures, and by that consideration to win a glimpse, as it were, or a shadow of perception, that your wisdom, might, and goodness are infinite and unspeakable, declared in your creatures; and being further advised by your merciful goodness that three principal means were used in the creation of all your creatures, namely number, weight, and measure; and since the two arts of number and measure (ancient, famous, and to human uses most necessary) are already sufficiently known and available — this third key we beseech of you (through your accustomed goodness) that it may come to the needed and sufficient knowledge of

such of your servants as would gladly find in your workmanship your true occasions (purposely designed) by which we should glorify your name, and show forth (to those weak in faith) your wondrous wisdom and goodness. Amen.”

Do not marvel at this outburst, godly friend, you gentle and zealous student. Another day, perhaps, you will perceive what moved me to it. Here, for now, I will give you some foundation, and with it some show of the benefits arising from this art. And because this art is rare, my words and practices might be too dark, unless some light were held before the matter; and that will best be done by giving you, out of Archimedes’ demonstrations, a few principal conclusions, as follows.

- ¹ The surface of every liquid, standing by itself and at rest, is spherical; its center is the same as the center of the Earth.
- ² If solid magnitudes of the same size or quantity as any liquid, and having the same weight, are lowered into that liquid, they will settle downward so that no part of them lies above the surface of the liquid; and yet for all that they will not sink utterly down, or drown.
- ³ If any solid magnitude lighter than a liquid is lowered into that same liquid, it will settle down into it so far that the quantity of that liquid equal to the part of the solid magnitude which has settled down into it is, in weight, equal to the weight of the whole solid magnitude.
- ⁴ Any solid magnitude lighter than a liquid, forced down into that same liquid, will move upward with a power equal to the amount by which the liquid of equal quantity to the whole magnitude is heavier than that same magnitude.
- ⁵ Any solid magnitude heavier than a liquid, lowered into that same liquid, will sink down utterly; and in that liquid it will be lighter by exactly the weight or heaviness of the liquid having size or quantity equal to the solid magnitude.
- ⁶ I. D. The cutting of a sphere according to any proportion assigned may by this proposition be done mechanically, by tempering a liquid to a certain weight in respect of the weight of the sphere swimming in it.

If any solid magnitude lighter than a liquid is lowered into that same liquid, the weight of that magnitude is to the weight of the liquid (which is equal in quantity to the whole magnitude) in the same proportion that the part of the magnitude settled down is to the whole magnitude.

By these truths, great errors may be corrected in the opinion of the natural motion of things, light and heavy. Which errors are allowed in natural philosophy by almost all men, from too much trust in authority and false assumptions; as, that of any two bodies, the heavier moves downward faster than the lighter.

A common error, noted.

This error was not first noted by me, but by one John Baptist de Benedictis. The chief of his propositions is this, which seems a paradox:

A paradox.

If there are two bodies of one form and of one kind, equal in quantity or unequal,

they will move through equal spaces in equal times: provided both their motions are in air, or both in water, or in any one medium.

Hereupon, in the practice of gunnery, certain good discourses may otherwise receive great amendment and furtherance.

The wonderful use of these propositions.

In the intended purpose also, allowing something to the imperfection of nature, which does not answer to the precision of demonstration. Moreover, by the foresaid propositions (wisely used) air, water, earth, and fire may be nearly known: how light or heavy they are (naturally) in their assigned parts, or in the whole. And then, turning your practice to elemental things, you may deal for the proportion of the elements in compounded things. Then, for the proportions of the humors in man: their weights, and the weight of his bones and flesh, etc. Then, by weight, to consider the force of a man in any manner, whole or in part. Then you may have pleasant consideration of the water-draft of ships, differing in the sea and in fresh water; and of weighing up anything sunk in sea or fresh water, etc. And (to lift your head aloft) by weight you may, as precisely as by any other instrument, measure the diameters of sun and moon, etc.

Friend, please weigh these things with the just balance of reason, and you will find wonders upon wonders; and esteem one drop of truth (yes, in natural philosophy) more worth than whole libraries of opinions undemonstrated, or not answering to nature's law and your experience. Leaving these things thus, I will give you two or three light practices to great purpose, and so finish my statical annotation. In mathematical matters, by the mechanician's aid, we will here see the benefit of weight.

The statical practice, to know the proportion between the cube and the sphere. Make a cube of one uniform material throughout, of like heavy stuff; and of the same stuff make a sphere or globe, precisely, of a diameter equal to the radical side of the cube. Your stuff may be wood, copper, tin, lead, silver, etc. (being, as I said, of like nature, condition, and like weight throughout). And you may, by assay balance, have prepared a great number of the smallest weights, which by that balance can be discerned or tried; and so have proceeded to make a perfect pile, company and number of weights, up to the weight of six, eight, or twelve pounds: all most diligently tried. And of every one, the content known, in your least weight that is weighable.

[Those who cannot have weights of precision may, with uniform and well-sifted sand, make themselves a number of weights somewhat near precision, by always halving the sand; they will at length come to a least common weight. There I leave the further matter to the discretion of those whom necessity shall press.]

The Venetian reckoning of weight may seem precise enough: by eight successive descents, halving from a grain.

I. D. For so you have 256 parts of a grain.

Your cube, sphere, suitable balance, and convenient weights being ready, fall to work. First, weigh your cube. Note the number of the weights. Then weigh your sphere, and likewise note the number of the weights. If you now find the weight of your cube to be to the weight of the sphere as 21 is to 11, then you see how the mechanician and experimenter, without geometry and demonstration, are (as nearly in effect) taught the proportion of the cube to the sphere, as I have demonstrated it at the end of the twelfth book of Euclid. Often try with

the same cube and sphere. Then change your sphere and cube to another material, or to another size, until you have made a perfect universal experience of it. It is possible that you may win to nearer terms in the proportion.

When you have found this one certain drop of natural truth, proceed on to infer, and duly to make trial of, the matter depending. For, since it is well demonstrated that a cylinder whose height and the diameter of its base are equal to the diameter of the sphere is sesquialter to that same sphere (that is, as 3 is to 2), to the number of the weight of the sphere add half as much again, and so you have the number of the weight of that cylinder. Which is also contained in our former cube: so that the base of that cylinder is a circle described in the square which is the base of our cube. But the cube and the cylinder, being both of one height, have their bases in the same proportion in which they are, one to another, in their bulk or solidity. But, beforehand, we have two numbers expressing their bulk, solidities, and quantities by weight; therefore, The proportion of the square to the circle inscribed.

we have the proportion of the square to the circle inscribed in the same square. And so we have fallen into the sensible and experimental knowledge of Archimedes' great secret, sought and found by him through great travail of mind. Therefore, to any circle given you can give a square equal:

The squaring of the circle, mechanically.

as I have taught in my annotation upon the first proposition of the twelfth book; and likewise to any square given you may give a circle equal:

To any square given, to give a circle equal.

If you describe a circle which shall be in that proportion to your inscribed circle as the square is to the same circle, this you may do by my annotations upon the second proposition of the twelfth book of Euclid, in my third problem there. Your diligence may come to a proportion of the square to the circle inscribed nearer the truth than is the proportion of 14 to 11. And consider that you may begin at the circle and square, and so come to conclude of the sphere and the cube what their proportion is, as now you came from the sphere to the circle. For, of silver or gold or brass sheets or plates (drawn through one hole, as is the manner) if you make a square figure and weigh it; and then, describing thereon

the inscribed circle, cut off and file away precisely (to the circle) the surplus of the square, you shall then, weighing your circle, see whether the weight of the square be to your circle as 14 to 11. As I have noted at the beginning of Euclid's twelfth book, etc. After this, resort to my last proposition upon the last of the twelfth; and there help yourself to the end. And here note this by the way.

Note: squaring the circle without knowledge of the proportion between circumference and diameter.

That we may square the circle without having knowledge of the proportion of the circumference to the diameter, as you have here perceived. And otherwise also, I can demonstrate it. So that many have encumbered themselves superfluously, by laboring in that point first which was not of necessity first, and also very intricate. And easily you may (and that variously) come to the knowledge of the circumference, the circle's quantity being first known. Which thing I leave to your consideration, making haste to dispatch another masterly problem, and to bring it nearer to your knowledge and readier dealing with than the world (before this day) had it for you, that I can tell of. And that is, a mechanical doubling of the cube, etc. Which may thus be done:

To double the cube readily, by mechanical art, depending upon mathematical demonstration.

Make of copper plates or tin plates a four-square upright pyramid, or a cone, perfectly fashioned hollow within. In this let great diligence be used to approach (as near as may be) to the mathematical perfection of those figures. At their bases let them be all open; everywhere else most close and just. From the vertex to the circumference of the base of the cone, and to the sides of the base of the pyramid:

I. D. The 4 sides of this pyramid must be 4 isosceles triangles alike and equal.

Let 4 straight lines be drawn in the inside of the cone and pyramid, making at their fall upon the perimeters of the bases equal angles on both sides of themselves with those perimeters. These 4 lines (in the pyramid, and as many in the cone) divide: one into 12 equal parts, another into 24, another into 60, and another into 100 (counting up from the vertex). Or use other numbers of division, as experience shall teach you.

I. D. In all workings with this pyramid or cone, let their situations be in all points and conditions alike, or all the same, while you are about one work. Else you will err.

Statics, 2



Then set your cone or pyramid, with the vertex downward, perpendicularly with respect to the base. (Though it is placed otherwise, it hinders nothing.) So let them be most steadily supported. Now, if there is a cube which you would have doubled, make a neat cube of copper, silver, lead, tin, wood, stone, or bone; or else make a hollow cube, or cubic coffer, of copper, silver, tin, or wood, etc. You may so proportion these with respect to your pyramid or cone that the pyramid or cone will be able to contain their weight in water 3 or 4 times over, at the least, whatever stuff they are made of. Let not your solid angle at the vertex be too sharp, but let the water come with ease to the very vertex of your hollow cone or pyramid. Put one of your solid cubes in a suitable balance; take its weight exactly in water. Pour that water (without loss) into the hollow pyramid or cone, quietly. Mark in your lines what numbers the water cuts at. Take the weight of the same cube again, in the same kind of water which you had before:

I. D. Consider well when you must put your waters together, and when you must empty your first water out of your pyramid or cone. Else you will err. put that also into the pyramid or cone where you put the first. Mark now again in what number or place of the lines the water cuts them. Two ways you may reach your conclusion: that is, either by numbers or by lines. By numbers: as, if you divide the side of your fundamental cube into as many equal parts as it is capable of, conveniently, with your ease and the precision of the division. For,

as the number of your first and lesser line (in your hollow pyramid or cone) is to the second or greater (both being counted from the vertex), so shall the number of the side of your fundamental cube be to the number belonging to the radical side of the cube double to your fundamental cube; which, being multiplied cubically, will soon show whether it is double the cubic number of your fundamental cube or not. By lines, thus: as your lesser and first line (in your hollow pyramid or cone) is to the second or greater, so let the radical side of your fundamental cube be to a fourth proportional line, by the 12th proposition of the sixth book of Euclid. Which fourth line shall be the cube root, or radical side, of the cube double to your fundamental cube; which is the thing we desired.

God be thanked for this invention and the fruit ensuing.

For this may I (with joy) say, EYPHKA, EYPHKA, EYPHKA, thanking the holy and glorious Trinity, having greater cause thereto than

Vitruvius, Book 9, Chapter 3.

Archimedes had, for finding the fraud used in the king's crown of gold; as all men may easily judge by the diversity of the fruit following from the one and the other.

Where I spoke before of a hollow cubic coffer, the like use is of it, and without weight. Thus: fill it with water precisely full, and pour that water into your pyramid or cone. And here note where the lines are cut in your pyramid or cone. Again fill your coffer, just as you did before. Put that water also to the first. Mark the second cutting of your lines. Now, as you proceeded before, so must you proceed here.

Note.

And if the cube which you should double be ever so great, you have thus the proportion (in small) between your two little cubes; and then the side of that great cube (to be doubled), being the third, will have the fourth found to it proportional, by the 12th of the sixth book of Euclid.

Note, concerning the spherical surface of the water.

Note that all this while I do not forget my first statical proposition, here rehearsed: that the surface of the water is spherical. In this use your discretion: to the first line adding a small hair's breadth more, and to the second half a hair's breadth more to its length. For you will easily perceive that the difference can be no greater in any pyramid or cone you are to handle. Which you shall thus try. For finding the swell of the water above level.

"Square the semidiameter from the center of the earth to the surface of your first water. Then square half the subtendent of that watery surface (which subtendent must have the equal parts of its measure all the same as those of the semidiameter of the earth to your watery surface). Subtract this square from the first; of the residue take the square root. Subtract that root from your first semidiameter of the earth to your watery surface: what remains is the height of the water in the middle above the level."

Which you will find to be a thing imperceptible. And though it were greatly perceptible,

Note.

yet, by help of my sixth theorem upon the last proposition of Euclid's twelfth book, you may reduce all to a true level. But further diligence is to be used by you against accidental causes of the water's swelling: as by having beforehand made moist your hollow pyramid or cone somewhat with a moist sponge, which will prevent an accidental cause of swelling, etc. Experience will teach you abundantly, with great ease, pleasure, and convenience.

Thus may you double the cube mechanically, treble it, and so forth, in any proportion.

Note this abridgment of doubling the cube, etc.

Now will I abridge your pain, cost, and care in this. Without all the preparing of your fundamental cubes, you may (alike) work this conclusion. For that was rather a kind of experimental demonstration than the shortest way, and all depending upon one mathematical demonstration. "Take water (as much as will conveniently serve your turn, as I warned before of your fundamental cube's size). Weigh it precisely. Put that water into your pyramid or cone. Of the same kind of water then take again the same weight you had before, and put

that likewise into the pyramid or cone. For each time, your marking of the lines, how the water cuts them, shall give you the proportion between the radical sides of any two cubes, of which the one is double to the other; working as before I have taught you:

Note.

saving that, for your fundamental cube's radical side, here you may take a straight line at pleasure."

Yet further proceeding with our drop of natural truth:

To give cubes one to another in any proportion, rational or irrational.

you may (now) give cubes one to another in any proportion given, rational or irrational, in this manner. Make a hollow parallelepiped of copper or tin, with one base wanting, or open, as in our cubic coffer. From the bottom of that parallelepiped raise up many perpendiculars in each of its four sides. Now if any proportion be assigned you in straight lines, cut one of your perpendiculars (or a line equal to it, or less than it) likewise, by the 10th of the sixth book of Euclid. And set those two parts in two separate lines of those perpendiculars (or you may set them both in one line), making their beginnings to be at the base, and so their lengths to extend upward. Now set your hollow parallelepiped upright, perpendicularly, steady. Pour in water to the height of your shorter line. Pour that water into the hollow pyramid or cone. Mark the place of the rise. Settle your hollow parallelepiped again. Pour water into it, to the height of the second line, exactly.

Emptying the first.

Pour that water duly into the hollow pyramid or cone; mark now again where the water cuts the same line which you marked before. For there, as the first marked line is to the second, so shall the two radical sides be, one to the other, of any two cubes which in their solidity shall have the same proportion which was at the first assigned, whether it be rational or irrational.

Thus, in sundry ways you may furnish yourself with such strange and profitable matter, which long has been wished for. And though it be done naturally and mechanically, yet it has a good mathematical demonstration.

The demonstrations of this doubling of the cube, and of the rest.

Which is this: always you have two like pyramids, or two like cones, in the proportions assigned; and like pyramids or cones are in proportion, one to another, in the proportion of their homologous sides (or lines) tripled. Therefore, if to the first and second lines found in your hollow pyramid or cone you join a third and a fourth in continual proportion, that fourth line shall be to the first as the greater pyramid or cone is to the lesser, by the 33rd of the eleventh book of Euclid. If pyramid to pyramid, or cone to cone, be double,

I. D. Hereby help yourself to become a precise practitioner. And so consider how you are hindered not at all (sensibly) by the convexity of the water.

then shall line to line be also double, etc. But, as our first line is to the second, so is the radical side of our fundamental cube to the radical side of the cube to be made or to be doubled; and therefore to those two also a third and a fourth line in continual proportion joined will give the fourth line in that proportion to the first as our fourth pyramidal or conic line was to its first; but that was double or triple, etc., as the pyramids or cones were one to another (as we have proved); therefore this fourth shall be also double or triple to the first, as the pyramids or cones were one to another. But our made cube is described of the second in proportion, of the four proportional lines:

By the 33rd of the eleventh book of Euclid.

therefore, as the fourth line is to the first, so is that cube to the first cube; and we have proved the fourth line to be to the first as the pyramid or cone is to the pyramid or cone. Therefore the cube is to the cube as pyramid is to pyramid, or cone is to cone.

I. D. And your diligence in practice can so (in weight of water) perform it; therefore, now, you are able to give good reason for your whole doing.

But we suppose pyramid to pyramid, or cone to cone, to be double or triple, etc. Therefore cube is to cube double, or triple, etc. Which was to be demonstrated. And of the parallelepiped it is evident that the solid water parallelepipeds are one to another as their heights are, seeing they have one base. Therefore the pyramids or cones made of those water parallelepipeds are one to another as the lines are (one to another) between which our proportion was assigned. But the cubes made of lines, after the proportion of the pyramidal or

conic homologous lines, are one to another as the pyramids or cones are one to another (as we before did prove); therefore the cubes made shall be one to another as the lines assigned are one to another. Which was to be demonstrated. Note.

Note this corollary.

This my demonstration is more general than only in a square pyramid or cone; consider well. Thus have I, both mathematically and mechanically, been very long in words; yet (I trust) nothing tedious to those who are well affected toward these things. And truly I am forced (avoiding prolixity) to omit sundry such things easy to be practiced, which to the mathematician would be a great treasure, and to the mechanician no small gain.

The great benefits following from these new inventions.

Now may you, between two lines given, find two middle proportionals in continual proportion, by the hollow parallelepiped and the hollow pyramid or cone. Now, any rectangular parallelepiped being given, three straight lines may be found proportional in any proportion assigned, of which shall be produced a parallelepiped equal to the parallelepiped given. Hereof I noted somewhat upon the 36th proposition of the 11th book of Euclid. Now, all those things which Vitruvius in his Architecture specified as able to be done by doubling of the cube, or by finding of two middle proportionals between two lines given, may easily be performed. Now, that problem which I noted to you at the end of my addition upon the 34th of the 11th book of Euclid is proved possible. Now may any regular body be transformed into another, etc. Now, any regular body: any sphere, yes, any mixed solid; and (what is more) irregular solids may be made (in any proportion assigned) like the body first given. Thus, of a mannequin (as the Dutch painters term it) in the same symmetry may a giant be made, and that with any gesture used by the mannequin, and conversely. Now may you, of any mold or model of a ship, make one of the same mold (in any assigned proportion) bigger or lesser.

Now may you, of any gun, or little piece of ordnance, make another, with the same symmetry (in all points) as great and as little as you will. Mark that, and think on it. Infinitely may you apply this, so long sought for and now so easily

concluded, and withal so willingly and frankly communicated to such as faithfully deal with virtuous studies.

Such is the fruit of the mathematical sciences and arts.

Statics, 3



Thus the mathematical mind can deal speculatively in its own art, and by good means mount above the clouds and stars; and thirdly, it can, in order, descend, to frame natural things to wonderful uses, and, when it pleases, retire home into its own center, and there prepare more means to ascend or descend by, all to the glory of God and our honorable delight on earth.

Although the printer has looked for this preface a day or two, yet I could not bring my pen from the paper before I had given you encouraging notice and brief instructions about some of the benefits that may be reaped by statics. In the rest I will therefore be as brief as I possibly can, and, at the same time, describe them somewhat accordingly. And you will perceive as much from what comes next in order. For, whereas it is so ample and wonderful that one might find in it fruitful matter to speak of for a whole year, and in practice is a treasure without end, yet I will glance over it with very few words.

Anthropography



This I call Anthropography: an art restored, and an advancement of mine in your service. I ask you to think of it as one of the chief points of human knowledge. Although it has only now first been confirmed with this new name, yet the matter has, from the beginning, been under consideration by all perfect philosophers. Anthropography is the description of the number, measure, weight, figure, situation, and color of every diverse thing contained in the perfect body of man, with certain knowledge of the symmetry, figure, weight, characterization, and due local motion of any assigned part of the said body, and of the numbers belonging to that part. This is the one part of the definition suited to this place, sufficient to show the particularity and excellency of the art, and why it is here ascribed to the mathematical sciences. If the description of the heavenly part of the world has a peculiar art called astronomy; if the description of the earthly globe has its peculiar art called geography; if the matching of both has its peculiar art called cosmography, which is the description of the whole and universal frame of the world; why should not the description of

Man is the lesser world.

him who is the lesser world, and from the beginning called Microcosmus (that is, the lesser world), and for whose sake and service all other bodily creatures were created, who also partakes with spirits and angels and is made in the image and similitude of God, have its peculiar art, and be called the art of arts, rather

than either lack a name or have too base and improper a one? You must borrow or claim back for yourself the peculiar parts of this from sundry professions, and proceed farther, as God, nature, reason, and experience shall inform you. The anatomists will restore to you some part; the physiognomists, some; the chiromantists, some; the metaposcopists, some; the excellent Albert Durer, a good part; the art of perspective will help the eye forward somewhat; Pythagoras, Hippocrates, Plato, Galen, Meletius, and many others, in certain things, will be contributors. And farther, the heavens, the earth, and all other creatures will each show and offer their harmonious service to fill up whatever is lacking hereof; and, concluding with your own experience, you may methodically register the whole for posterity, by which good proof will be had of our harmonious and

The microcosm.

microcosmic constitution.

The outward image and view of this is most necessary and profitable to the art of drawing and painting, to sculpture, and to architecture (for church, house, fort, or ship), for it is the chief base and foundation of them.

Book 3, chapter 1.

Look in Vitruvius, whether I deal sincerely for your benefit or not. Look in Albertus Durerus, *On the Symmetry of the Human Body*. Look in chapters 27 and 28 of the second book, *On Occult Philosophy*. Consider the Ark of Noah. And from that, wade farther. Remember the Delphic oracle “Know yourself”, pronounced so long ago, repeated by so many a philosopher, and attempted by the wisest; and then you will perceive how long ago you have been called to the school where this art might be learned. Well, I am not at all afraid of the disdain of some who think the sciences and arts to be only seven. Perhaps such people may, with ignorance and shame enough, come short of those seven as well; and yet nevertheless they cannot set down a certain number of arts, nor in each art certain impassable bounds for God, nature, and human industry. New arts rise up daily; and no such order was taken that all the arts should be made known to the world in one age, or in one land, or by one man. Let us embrace the gifts

of God and the ways to wisdom, in this time of grace, continually bestowed from above on those who will receive them thankfully: and to the good, all things shall work together for good.

Trochilike



Trochilike is that mathematical art which demonstrates the properties of all circular motions, simple and compound. And because the fruit of it, commonly received, is in wheels, it has the name Trochilike, as one might say, Wheel Art. By this art a wheel may be given which shall turn once about in any assigned time. Two wheels may be given whose turnings about in one and the same time (or in equal times) shall have to one another any appointed proportion. By wheels a straight line may be described; likewise a spiral line in a plane, conic section lines, and other irregular lines may be drawn at pleasure. These and such like are principal conclusions of this art, and help forward many pleasant and profitable mechanical works:

Sawmills.

Mills that saw great and very long deal boards, with no man standing by. Such I have seen in Germany, and in the city of Prague, in the kingdom of Bohemia; coining mills, hand mills for grinding corn; and all manner of mills and wheelwork, moved by wind, smoke, water, weight, spring, man or beast. Take up in your hand Agricola's *On the Nature of Metals*, and then you shall perceive, in all mines, how great need there is of wheelwork. By wheels strange and incredible works are done, as will appear hereafter in other arts. A wonderful example of further possibility and of present benefit was seen in my time in a certain instrument: which its inventor and artificer had formerly sold for twenty talents of gold, and which had then, by misfortune, suffered some injury and

hurt. One Ianellus of Cremona mended it, and presented it to Emperor Charles the Fifth. Hieronymus Cardanus can be my witness that in it there was one wheel which moved, and that at such a rate that in 7000 years only its own period would be completed. A thing almost incredible; but how far I keep within my bounds, very many men yet alive can tell.

Helicosophy



Helicosophy is near sister to Trochilike, and is a mathematical art which demonstrates the drawing of all spiral lines, in a plane, on a cylinder, cone, sphere, conoid, and spheroid, and the properties belonging to them. Its use in architecture and in diverse instruments and engines is most necessary. For in many things the screw performs the feat which otherwise could not be accomplished. By the help of this,

Athenaeus, book 5, chapter 8.

it is recorded that, whereas all the power of the city of Syracuse could not move a certain ship (being on the ground), mighty Archimedes, applying his screw engine, caused Hiero the king, by himself and at ease, to move her just as he pleased.

Proclus, page 18.

At this the king, wondering, said: *Απὸ ταύτης τῆς ἡμέρας, περὶ παντὸς, Αρχιμήδει λέγοντι πισευτεόν.* “From this day forward (said the king), credit ought to be given to Archimedes, whatever he says.”

Pneumatithmie



Pneumatithmie demonstrates, by closed hollow geometric figures (regular and irregular), the strange properties (in motion or rest) of water, air, smoke, and fire in their continuity, and as they are joined to the elements next them. This art is very profitable to the natural philosopher, to prove that a vacuum, or emptiness, is not in the world; and that all nature abhors it so much that, contrary to ordinary law, the elements will move or stand. Thus water will ascend, rather than leave between it and the air a space or place more than that quantity of air naturally requires or can fill. Again, water will hang and not descend, rather than leave emptiness behind it by descending. The same is true of fire and air: they will descend rather than have their continuity dissolved, or their next element forced away from them. And as they will not be extended to discontinuity, so they will not, nor can they by any human force, be pressed or pent into a space not sufficient and answerable to their bodily substance. Great force and violence will they use to enjoy their natural right and liberty.

To go to the bottom of the sea without danger.

Hereupon, two or three men together, by keeping air under a great cauldron and forcing it down in order, may descend without harm to the bottom of the sea, and stay there for a time, etc. Where note how the thicker element, as water, gives place to the thinner, as is the air, and receives violence from the thinner in like manner, etc. Pumps and all manner of bellows have their ground in this art, and many other strange devices; as hydraulica, organs

worked by water, etc. Of this feat, commonly called Pneumatica, goodly works are extant, both in Greek and Latin. With old and learned schoolmen it is called the Science of the Full and the Empty.

Menadrie



Menadrie is a mathematical art which demonstrates how, above nature's simple virtue and power, virtue and force may be multiplied, and so direct, lift, pull toward, and push or cast from, any multiplied or simple determined virtue, weight, or force that is not naturally so easy to direct or move. This art is furthered very much by other arts: in some points by perspective; in some by statics; in some by Trochilike; and in others by Helicosophy and Pneumatithmie. By this art all cranes, gibbets, and engines to lift up or to force anything, in any manner of way, are ordered, and the certain cause of their force is known: as the force that one man has with the Dutch wagon rack, with which to set up again a mighty laden wagon that has been overturned. The force of the crossbow rack is certainly here demonstrated. The reason why one man with a lever lifts that which six men with their hands only could not so easily do. By this art, in our common cranes in London, where the power is to crane up a weight of 2000 pounds, by two more wheels (added by good ordering) the art concludes that there may be craned up a weight of 200000 pounds, etc. So well did Archimedes know this art that he alone, with his devices and engines, twice or thrice spoiled and discomfited the whole army and host of the Romans besieging Syracuse,

Plutarch in Marco Marcello.

Marcus Marcellus the Consul, being their commander-in-chief.

Synesius, in his Letters.

Such huge stones, so many, with such force, and so far, did he with his engines hail among them out of the city.

Polybius.

Pliny.

Quintilian.

T. Livius.

And by sea likewise, though their ships might come to the walls of Syracuse, yet he utterly confounded the Roman navy. What with his mighty stones hurling:

Athenaeus.

what with pikes 18 feet long, made like shafts, which he forced almost a quarter of a mile; what with his catching hold of their ships, hoisting them up above the water, and suddenly letting them fall into the sea again;

Galen.

Anthemius.

what with his burning glasses, by which he fired their other ships afar off; what with his other policies, devices, and engines, he so manfully acquitted himself that all the force, courage, and policy of the Romans (for a great season) could nothing prevail for the winning of Syracuse. After which the Romans named Archimedes Briareus and Centimanus. Zonaras makes mention of one Proclus, who so well had perceived Archimedes' art of Menadrie, and had so well invented on his own, that with his burning glasses,

Burning glasses.

being placed upon the walls of Byzantium, he multiplied the heat of the sun so, and directed its beams against his enemies' navy with such force and so suddenly (like lightning), that he burned and destroyed both man and ship. And Dion specifies of Priscus, a geometrician in Byzantium, who invented and used sundry engines of force multiplied; which was the cause that Emperor Severus pardoned him his life after he had won Byzantium, because he honored the art, wit, and rare industry of Priscus. But nothing inferior to the invention of these engines of force was the invention of guns.

Guns.

Which took its occasion and order of first invention from an Englishman, though in another land and by other men it was first carried out. And those who should see the record where the occasion and general order of gunning is first discussed would think that “small things, slight and common, coming to wise men’s consideration and industrious men’s handling, may grow to be of force incredible.”

Hypogeiodie



Hypogeiodie is a mathematical art demonstrating how, under the spherical surface of the earth, at any depth, to any assigned perpendicular line (whose distance from the perpendicular of the entrance, and the azimuth likewise with respect to the said entrance, are known), a certain way may be prescribed and traveled; and how any way designed above the surface of the earth may be kept under ground at any limited depth, always going perpendicularly beneath the way designed on the earth; and, conversely, given any way (straight or crooked) under the earth, to lay it out upon the outer face or surface of the earth, so that perpendiculars drawn from the center of the earth to the spherical surface of the earth shall precisely fall in the corresponding points of those two ways. This, with all other cases and circumstances in this, and appurtenances, this art demonstrates. This art is very ample in variety of conclusions, and very profitable in sundry ways to the commonwealth. The occasion of my inventing this art was at the request of two gentlemen who had a certain work (of gain) under ground, and their grounds joined over the work; and by reason of the crookedness, diverse depths, and heights of the way under ground, they were in doubt, and at controversy, over under whose ground the work then was. The name only (before this) was published by me, *On the Subterranean Journey*; the rest is at God's will. For pioneers, miners, diggers for metals, stone, and coal, and for

secret passages under ground between place and place (as this land has diverse), and for other purposes, any man may easily perceive both the great fruit of this art, and also in this art the great aid of geometry.

Hydragogie



Hydragogie demonstrates the possible leading of water, by nature's law and by artificial help, from any head (being a spring, standing, or running water) to any other place assigned. Long has this art been in use, and much of it written, and very marvelous works performed in it, as may still appear in Italy by the ruins remaining of the aqueducts; in other places, of rivers led through the mainland, navigable many a mile; and in other places, of the marvelous forcings of water to ascend. All these declare the great skill that must be required of one who would be perfect in this art for all possible occasions of leading water. I need not speak of the allowance of the fall for every hundred feet, or of the vents (if the water's labor be far and great), seeing that at hand (about us) many expert men can sufficiently testify to the effect of the order, though they do not know the demonstration of its necessity; nor yet, if they were led up and down and about mountains from the head of the spring, and then a place being assigned, and them being asked how low or high that last place is with respect to the head from which (so crookedly and up and down) they have come, perhaps they would not or could not very readily or precisely answer that question. Geometry is therefore necessary to Hydragogie. Of the sundry ways to force water to ascend, whether by tympanum, kettle mills, screw, Ctesibike, or such like, the manner may fully appear in Vitruvius, Agricola, and others; and

so by that also may be most evident how the arts of Pneumatithmie, Helicosophy, statics, Trochilike, and Menadrie come to the furnishing of this in speculation, and to the benefit of the commonwealth in practice.

Horometry



Horometry is a mathematical art that demonstrates how, at any appointed time, the precise customary reckoning of time may be known for any given place. These words are smooth, plain, easy English, but their meaning reaches farther than you would readily imagine. Some part of this art was called in old times Gnomonice, and more recently Horologiographia, and in English it may be called dialing. The use is ancient, and the invention is older still. The use plainly appears to have been in existence, at least, more than two thousand three hundred years ago:

(4. Reg. 20.)

In King Achaz's dial, then, by the sun, showing the distinction of time. By sun, moon, and stars this dialing may be carried out, and the precise time of day or night known. But the demonstrative drawing of dials of every sort requires good skill in astronomy and in elemental, spherical, phenomenal, and conic geometry. Then, to apply the grounds of the art to any regular surface offered in any place, and there to describe, in every manner of way, how the usual hours may truly be determined by the sun's shadow, will be found no slight work for a draftsman: so to draw and prescribe the sun's motion to the breadth of a hair. In this feat (in my youth) I invented a way how, in any horizontal, mural, or equinoctial dial, etc., at all hours (the sun shining) the ascending sign and degree may be known. Which is a thing very necessary for the rising of those fixed stars whose operation in the air is plainly of great power. I say no more of

its use here. But since human affairs require knowledge of times and moments when neither sun, moon, nor star can be seen, mechanical industry first invented how, by water running in orderly course, the time and hours might be known, of which the famous Ctesibius was the inventor, a man whom Vitruvius justly extolled to the sky. After that, hours were measured by running sand; then by Trochilike driven by weight; and in recent times by Trochilike driven by a spring, without weight. All of these, directed by sun or stars at certain times, require oversight and correction according to the heavenly equinoctial motion, besides the inequality of their own working. There remains (without any parabolic meaning here) among the philosophers,
(A perpetual motion.)

a more excellent, more useful, and more marvelous way than all these: to have the motion of the Primouant (the first equinoctial motion) imitated by nature and art, which you shall hereafter understand more of through further search in weightier studies. And so it is time to finish this account of the division of time used in our common and private affairs, the benefit of which no man would want who knows how to spend his time.

Zography



Drawing is a mathematical art which teaches and demonstrates how the intersection of all visual pyramids, made by any assigned plane (the center, distance, and lights being determined), may be represented by lines and proper colors. A notable art this is, and it would require a whole volume to declare its nature and the benefits ensuing from it. Great skill of geometry, arithmetic, perspective, and anthropography, with many other particular arts, the draftsman needs for his perfection. For the most excellent painter (who is but the proper mechanic and the sensible imitator of the draftsman) has attained to such perfection that the senses of men and beasts have judged painted things to be natural and not artificial, alive and not dead. This mechanical draftsman (commonly called the painter) is marvelous in his skill, and seems to have a certain divine power: to make of absent friends a friendly, present comfort; yes, and of dead friends to give a continual, silent presence, not only with us but with our posterity for many ages.

And so proceeding, consider: in winter he can show you the lively view of summer's joy and riches, and in summer present the face of winter's doleful state and nakedness. Cities, towns, forts, woods, armies, yes whole kingdoms (however distant or great) he can easily bring home with him (into anyone's judgment) as living patterns of the things rehearsed. In one little house he can enclose (to the great pleasure of the beholders) the lively portrait of all visible creatures: those on earth or living under the earth; or in the waters, lying, creep-

ing, sliding, or swimming; or any bird or fly flying in the air. Even set against the stars, the sky, the clouds, yes, in the very appearance of light itself (that divine creature), can he match the judgment of our eyes most nearly. What a thing this is! Things that do not yet exist he can represent so that, when they do exist, the picture seems (in a manner) to have created them. To what craftsman is painting not a great pleasure and benefit? Which of them all would refuse the direction and aid of painting? The architect, the goldsmith, and the tapestry weaver hold painting in high regard. Our vivid herbals, our portraits of birds, beasts, and fishes, and our elaborate anatomies, in what way are they most perfectly made, or beheld by us with most pleasure? Is it not by painting alone? And if painting, by the painter's industry, is so beneficial and marvelous, what shall be thought of drawing, the schoolmaster of painting and its chief governor? Though I do not mention sculpture in my table of mathematical arts, yet all men may perceive how painting and sculpture are full sisters, and both very profitable in a commonwealth. And of sculpture, as much as of painting, excellent craftsmen have written large books in its praise. I take as witnesses Georgio Vasari, Pittore Aretino; of Pomponius Gauricus; and others. To these two arts (and others) a certain odd art called Althalmasat owes much, more than the ordinary sculptor, carver, cutter, engraver, founder, or painter, etc., knows his art to owe.

Architecture



(An objection.)

Architecture seems to many not worthy, or not fit, to be reckoned among the mathematical arts. To them I think it good to give some account of my doing so. Not worthy, they will say, because it is only for the building of a house, palace, church, fort, or such like gross works. And you also defined the mathematical arts to be such as dealt with no material or corruptible thing, and also did demonstratively proceed in their faculty by number or magnitude. First,

(The answer.)

You see that I count Architecture here among those mathematical arts which are derived from the principal arts; and you know that such may deal with natural things and sensible matter. Of which, “some draw nearer to simple and absolute mathematical speculation than others do.

And though the architect procures, informs, and directs the mechanic in the handwork and the actual building of house, castle, or palace, and is chief judge of the same, yet with himself (as chief master and architect) remains the demonstrative reason and cause of the mechanic’s work: in line, plane, and solid; by geometric, arithmetic, optic, musical, astronomical, cosmographic” (to be brief) by all the former derived mathematical arts and other natural arts, able to be confirmed and established. If this be so, then you may think that Architecture has good and due allowance in this honest company of derived mathematical arts. I will here crave the judgment of two most perfect archi-

fects: the one, Vitruvius the Roman, who wrote ten books of it to the Emperor Augustus (in whose days our heavenly Archemaster was born); and the other, Leo Baptista Albertus, a Florentine, who also published ten books of it. Architecture (says Vitruvius) is a science garnished with many doctrines and diverse instructions, by whose judgment all works finished by other workmen are judged. It follows: Architecture grows of framing and reasoning, etc. Reasoning is that which, of things framed with forecast and proportion, can make demonstration and manifest declaration. Again: In all things, therefore chiefly in Architecture, these two things are: the thing signified, and that which signifies. The thing propounded, of which we speak, is the thing signified. But demonstration, expressed with the reasons of diverse doctrines, signifies the same thing. After that: An architect (says he) ought to understand languages, to be skilled in painting, well instructed in geometry, not ignorant of perspective, furnished with arithmetic, to have knowledge of many histories, and diligently to have heard philosophers, to have skill in music, not ignorant of medicine, to know the answers of lawyers, and to have astronomy and the courses of the heavens in good knowledge. He gives orderly reason why all these arts, doctrines, and instructions are requisite in an excellent architect. And (for brevity) omitting the Latin text, thus he has it. Secondly, it is helpful for an architect to have the knowledge of painting: that he may the more easily fashion out, in painted patterns, the form of whatever work he likes. And geometry gives to architecture many helps: and first teaches the use of the rule and the compass, by which (chiefly and easily) the descriptions of buildings are dispatched in ground plans, and the directions of squares, levels, and lines. Likewise, by perspective, the lights of the sky are well led in buildings from certain quarters of the world. By arithmetic the charges of buildings are summed together, the measures are expressed, and the hard questions of symmetries are discoursed on by geometric means and methods, etc. Besides this, of the nature of things (which in Greek is called *φυσιολογία*) philosophy makes declaration. Which it is necessary for an architect diligently to have learned, because it has many and diverse natural questions, as especially in aqueducts. For in their courses, their leadings about in the level ground and in the mountings, the natural spirits or

breaths are engendered diverse ways. The hindrances which they cause no man can help, but he who out of philosophy has learned the original causes of things. Likewise, whoever shall read Ctesibius or Archimedes' books (and others who have written such rules) cannot think as they do, unless he shall have received from philosophers instruction in these things. And he must know music too, that he may have understanding both of regular and mathematical music: that he may tune well his ballistas, catapults, and scorpions, etc. Moreover, the bronze vessels which in theaters are placed by mathematical order in niches beneath the steps, and the diversities of the sounds (which the Greeks call *ἡχεῖα*) are ordered according to musical symphonies and harmonies, being distributed through the compass by Diatessaron, Diapente, and Diapason: so that the fitting voice of the players' sound, coming to these preparations made in order and there being increased, might with that increase come more clear and pleasant to the ears of the listeners, etc. And by astronomy are known the east, west, south, and north; the fashion of the sky, the equinox, the solstice, and the course of the stars. Which things, unless one knows, he cannot perceive anything at all of the reason of horologies. Seeing therefore this ample science is garnished, beautified, and stored with so many and sundry skills and knowledges, I think that none can justly account themselves architects all at once. But only those who, from their childhood years, ascending by these degrees of knowledge, being fostered up with the attainment of many languages and arts, have won their way to the high tabernacle of architecture, etc. And to whom nature has given such quick discernment, sharpness of wit, and memory that they may be thoroughly skilled in geometry, astronomy, music, and the rest of the mathematical arts: such surpass and pass the calling and state of architects:

(A mathematician.)

and have become mathematicians, etc. And they are found seldom. As in times past was Aristarchus Samius; Philolaus and Archytas, Tarentines; Apollonius Pergaeus; Eratosthenes Cyreneus; Archimedes and Scopas, Syracusians. Who also left to their posterity many engines and gnomonical works, invented and declared by numbers and natural means.

Thus much, and the same words (in sense) in one single chapter of this incomparable architect Vitruvius shall you find. And if you should but take his book in hand and look through it ever so slightly, you would say at once:

(Vitruvius.)

This is geometry, arithmetic, astronomy, music, anthropography, Hydragogie, horometry, etc. And (to conclude) the storehouse of all workmanship. Now let us listen to our other judge, our Florentine Leo Baptista, and narrowly consider how he determines of architecture. But before I proceed any further (says he) I think that I ought to express what man I would have allowed an architect. For I will not bring in place a carpenter, as though you might compare him to the chief masters of other arts. For the hand of the carpenter is the architect's instrument.

(Who is an architect.)

But I will appoint the architect to be "that man who has the skill (by a certain and marvelous means and way) both in mind and imagination to determine, and also in work to finish, whatever works, by motion of weight and the coupling and framing together of bodies, may most aptly be made useful for the worthiest uses of man." And that he may be able to perform these things he needs attainment and knowledge of the best and most worthy things, etc. The whole practice of architecture in building consists in lineaments and in framing. And the whole power and skill of lineaments tends to this: that the right and absolute way may be had of adapting and joining lines and angles, by which the face of the building or frame may be comprehended and concluded. And it is the property of lineaments to prescribe to buildings and every part of them a fitting place and certain number: a worthy manner and a seemly order: that so the whole form and figure of the building may rest in the very lineaments, etc. And we may prescribe in mind and imagination the entire forms, (The immateriality of perfect architecture.)

all material stuff being excluded. Which point we shall attain by noting and forepointing the angles and lines by a sure and certain direction and connection. Seeing then that these things are thus:

(What lineament is.)

Lineament shall be the certain and constant prescribing, conceived in mind: made in lines and angles: and finished with a learned mind and wit. “We thank you, Master Baptist, that you have so aptly brought your art and the phrase of it to a certain mathematical perfection:

by certain order, number, form, figure, and mental symmetry:” all natural and sensible stuff set aside. Now, then, it is evident, gentle reader, how aptly and worthily I have preferred Architecture to be bred and fostered in the dominion of the peerless princess Mathematics, and to be a natural subject of hers. And the name of Architecture comes from the principality which this science has above all other arts. And Plato affirms the architect to be master over all who make any work. So he is neither smith nor builder, nor separately any craftsman, but the head, provost, director, and judge of all artificial works and all craftsmen. For the true architect is able to teach, demonstrate, distribute, describe, and judge all works wrought. And he alone searches out the causes and reasons of all artificial things. Thus excellent is architecture, though few in our days attain to it; yet the art must not be thought of as anything less than it truly deserves. Nor may we make new and imperfect definitions of ancient arts in our days, for scarcity of craftsmen: no more than we may cut short the definitions of wisdom, or honesty, or friendship, or justice. Neither will I consent to diminish any whit of the perfection and dignity (by just cause) allowed to absolute architecture. Under the direction of this art are three principal, necessary mechanical arts. Namely housing, fortification, and shipbuilding. By housing I understand buildings both for divine service and human common use, public and private. Of fortification and shipbuilding strange matter might be told you; but perhaps some will be tired with this long list already rehearsed, and others will finely pick apart my coarse, homely discourse written in haste, for fear you should lack this true and friendly warning, and this foretaste of mathematical power. Life is short and uncertain; times are perilous, etc. And with the printer still waiting while my pen lingers: all these things, with further matter of ingratitude, give me occasion to move on to the other arts remaining, with all speed possible.

Navigation



The art of Navigation demonstrates how a sufficient ship, between any two places assigned (in navigable passage), may be conducted by the shortest good way, by the aptest direction, and in the shortest time; and in all storms and natural disturbances that may happen, how to use the best possible means to recover the place first assigned. What need the master pilot has of the other arts recited before is easy to know: of hydrography, astronomy, astrology, and horometry; always presupposing the common base and foundation of all, namely arithmetic and geometry. So that he is able to understand and judge his own necessary instruments and necessary equipment, whether they are perfectly made or not, and also can, if need be, make them himself: quadrants, the astronomer's ring, the astronomer's staff, the universal astrolabe, a hydrographical globe, true hydrographical charts (not with parallel meridians), the common sea compass, the compass of variation, the proportional and paradoxical compasses.

(Year 1559.)

(Invented by me for our two Moscouy master pilots, at the request of the Company) clocks with springs: hour, half-hour, and three-hour sand glasses, and sundry other instruments; and also to be able, on globe or plane, to describe the paradoxical compass, and duly to use the same to all manner of purposes for which it was invented; and also to be able to calculate the planets' places for all times.

Moreover, with sun, moon, or star (or without them) he is able to define the longitude and latitude of the place which he is in; so that the longitude and latitude of the place from which he sailed be given, or known to him. To which belong expert means of always being certain of the ship's course, and so on. And by foreseeing the rising, setting, culmination at noon, or culmination at midnight of certain tempestuous fixed stars, or their conjunctions and angles with the planets, and so on, he ought to have expert conjecture of storms, tempests, and waterspouts, and such like meteorological effects, dangerous at sea. For, as Plato says, To foresee the alterations and opportunities of times is fitting no less to the art of war than to husbandry and navigation. And besides such skillful means, more evident tokens in sun and moon ought to be known to him: such as the philosophical poet Virgilius teaches in his Georgics, where he says:

(Georgic. 1.)

The sun also, both at his rising and when he sinks into the waves, will give signs; the surest signs follow the sun, etc. ... For often we see that on his very face various colors wander. Blue, he announces rain; fiery, the east winds. But if spots begin to mingle with the ruddy fire, then you will see everything alike seething with wind and storm clouds: on such a night let no one urge me to go out over the deep, or to pull up the cable from the land, etc. The sun will give you signs. Who would dare call the sun false? ... etc.

And so of moon, stars, water, air, fire, wood, stones, birds, and beasts, and of many things else, a certain sympathetic forewarning may be had, sometimes to great pleasure and profit, both on sea and land. Enough for my present purpose it appears from the premises how mathematical the art of navigation is, and how it needs and also uses other mathematical arts. And now, if I should go about to speak of the many benefits coming to this land and others by ships and navigation, you might think that I catch at occasions to use many words where none are needed.

Yet this one thing may I justly say: in navigation, none ought to take greater care to be skillful than our English pilots. And perhaps some would attempt more, and others more willingly would help, if they knew for certain what priv-

ilege God had endowed this island with, by reason of a situation most convenient for navigation to places most famous and rich. And though,

(Year 1567 S. H. G.)

(Of late) a young gentleman, a courageous captain, was in great readiness, with good hope and strong grounds of confidence, to have ventured on a discovery (either westward, by Cape de Paramantia; or eastward, above Nova Zemla and the Cyremisses), and was, at the very near time of attempting it, called and employed otherwise, both then and since, in great good service to his country, as the Irish rebels have experienced:

(Year 1569)

Yet I say, though the same gentleman does not hereafter deal with the matter, someone or other should attend to it; and by good advice and careful discretion, little by little, gain sufficient knowledge of that trade and voyage; which now I would be sorry, through carelessness, lack of skill and courage, should remain unknown and unheard of. Seeing also that, in this matter, we are half challenged by the learned and half requested to publish. From this, truly, might grow benefit, to this land chiefly, and to the rest of the Christian commonwealth, far surpassing all riches and worldly treasure.

Thaumaturgike, 1



Thaumaturgike is that mathematical art which gives a certain order to make strange works, to be perceived by the sense, and greatly to be wondered at by men. By sundry means this wonder-work is wrought: some by Pneumatithmie, as the works of Ctesibius and Hero; some by weight, of which Timæus speaks; some by strings strained or springs, by that imitating lively motions; some by other means, as the images of Mercurie, and the brazen head made by Albertus Magnus which seemed to speak. Boethius was excellent in these feats. To whom Cassiodorus, writing, says, “Your purpose is to know profound things and to show marvels. By the disposition of your art, metals bellow: Diomedes of brass blows a trumpet loud: a brazen serpent hisses: made birds sing sweetly. Small things we rehearse of you, who can imitate heaven, and so on.” Of the strange self-moving which, at Saint Denys by Paris,

(Year 1551)

I saw, once or twice (Orontius being then with me, in company); it would be too strange to tell. But some have written it; and yet, I hope, it is there to be seen by others. And by perspective also strange things are done, as partly before I gave you to understand in perspective: as, to see in the air aloft the lively image of another man, either walking to and fro or standing still; likewise, to come into a house and there see the lively show of gold, silver, or precious stones, and

coming to take them in your hand, find nothing but air. By this have some men (in all other matters counted wise) badly overshot themselves, misjudging the means. Therefore said Claudius Cælestinus:

(On those things that happen marvelously in the world. Chapter 8.)

Nowadays we see some men, yes of great learning and reputation, judge certain works to be marvelous, above the power of nature; of which works one skilled in perspective might easily have given the cause. Of Archimedes' sphere, Cicero bears witness.

(Tusculan Disputations 1.)

Which is very strange to think on. For when Archimedes, says he, fastened in a sphere the movements of the sun, moon, and of the five other planets, he did as the god who (in *Timæus* of Plato) made the world: that, with one turning, he should rule motions most unlike in slowness and swiftness. But a greater cause for wondering we have from Claudianus' report of this, who affirms this work of Archimedes to have been of glass, and discourses of it more at length, which I omit. The dove of wood, which the mathematician Archytas made to fly, is spoken of by Agellius. Of Dædalus' strange images Plato reports. Homere, of Vulcan's self-movers (by secret wheels), leaves in writing. Aristotle, in his *Politics*, makes mention of both. Marvelous was the workmanship, in late days, performed by good skill of Trochilike, and so on. For in Noremberge a fly of iron, let out of the artisan's hand, did, as it were, fly about among the guests at the table, and at length, as though it were weary, return to its master's hand again. Moreover, an artificial eagle was ordered to fly out of the same town a mighty way, and that aloft in the air, toward the emperor coming there, and followed him when he had come to the gate of the town.

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Thus you see what the mathematical art can perform, when skill, will, industry, and ability are duly applied to proof.

(A Digression.)

And for these, and such like marvelous acts and feats, naturally, mathematically, and mechanically wrought and contrived:

(Apologeticall.)

ought any honest student and modest Christian philosopher be counted and called a conjurer? Shall the folly of idiots and the malice of the scornful so much prevail, that he who seeks no worldly gain or glory at their hands, but only from God the treasure of heavenly wisdom and knowledge of pure truth, shall he, I say, in the meantime be robbed and despoiled of his honest name and fame? He who seeks (by S. Paul's admonition) in the creatures' properties and wonderful virtues to find just cause to glorify the eternal and almighty Creator by: shall that man be (in secret) condemned as a companion of the hellhounds, and a caller and conjurer of wicked and damned spirits? He who bewails his great want of time, enough (to his satisfaction) to learn godly wisdom and godly truths in, and sets all his delight only in that: will that man lose and abuse his time in dealing with the chief enemy of Christ our Redeemer: the deadly foe of all humankind, the subtle and impudent perverter of godly truth, the hypocritical crocodile, the envious basilisk, continually desirous, in the twinkling of an eye, to destroy all humankind, both body and soul, eternally? Surely (for my part, somewhat to say in this) I have not learned to make so brutish and so wicked a bargain. Should I, for my twenty or twenty-five years of study: for two or three thousand marks spent: for seven or eight thousand miles gone and traveled, only for good learning's sake; and that in all manner of weather: in all manner of ways and passages: both early and late: in danger of violence by man: in danger of destruction by wild beasts: in hunger: in thirst: in perilous heat by day, with toil on foot: in dangerous damp colds by night, almost depriving life (as God knows): with lodgings often to little ease, and sometimes to less security; and for much more than all this done and suffered, for learning and the attainment of wisdom: should I, I ask you, for all this, no otherwise nor more warily (or by God's mercy no more luckily), have fished with so large and costly a net, so long a time in drawing (and that with the help and advice of Lady Philosophy and Queen Theology), but at length to have caught and drawn up a frog?

(A proverb. Fair fished, and caught a frog.)

No, a devil? For, so, does the common peevish prattler imagine and jangle; and so does the malicious scorner secretly wish, and bravely and boldly face down behind my back. Ah, what a miserable thing is this kind of men! How great is the blindness and boldness of the multitude in things above their capacity? What a land: what a people: what manners: what times are these? Have they become devils themselves, and, by false witness bearing against their neighbor, would they also become murderers? Does God so long give them respite, to reclaim themselves from this horrible slandering of the guiltless, contrary to their own consciences, and yet will they not cease? Does the innocent forbear calling them to answer him juridically, according to the rigor of the laws, and will they despise his charitable patience? As they, against him, by name, forge, fable, rage, and raise slander by word and print, will they provoke him, by word and print likewise, to note their names to the world, with their particular devices, fables, beastly imaginations, and unchristian-like slanders? Well, well. O you who are such, my unkind countrymen: O unnatural countrymen: O unthankful countrymen: O brainsick, rash, spiteful, and disdainful countrymen. Why do you oppress me so violently with your slandering of me, contrary to truth and contrary to your own consciences? And I, to this hour, neither by word, deed, or thought, have been in any way harmful, damaging, or injurious to you or yours? Have I so long, so dearly, so far, so carefully, so painfully, so dangerously sought and traveled for the learning of wisdom, and the attaining of virtue; and in the end (in your judgment) have I become worse than when I began? Worse than a madman? A dangerous member of the commonwealth, and no member of the church of Christ? Is this what you call being learned? Is this what you call being a philosopher, and a lover of wisdom? To forsake the straight heavenly way, and to wallow in the broad way of damnation? To forsake the light of heavenly wisdom, and to lurk in the dungeon of the prince of darkness? To forsake the truth of God and his creatures, and to fawn upon the impudent, crafty, obstinate liar, and continual disgracer of God's truth, to the utmost of his power? To forsake the life and bliss eternal, and to cleave to the author of everlasting death: that murderous tyrant most greedily awaiting the prey of a human soul? Well, I thank God and our Lord Jesus Christ, for the

comfort which I have by the examples of other men before my time: to whom, neither in godliness of life nor in perfection of learning, I am worthy to be compared; and yet they sustained the very same injuries that I do, or rather greater. Patient Socrates, his Apology, will testify: Apuleius, his Apologies, will declare the brutishness of the multitude. Ioannes Picus, Earl of Mirandula, his Apology will teach you of the raging slander of the malicious ignorant against him. Ioannes Trithemius, his Apology will specify how he had occasion to make public protestation, as much by reason of the rude and simple, as also in respect of those who were counted to be of the wisest sort of men. "Many could I recite: but I defer the precise and determined handling of this matter, being loth to detect the folly and malice of my native countrymen.

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Who so hardly can digest or like any extraordinary course of philosophical studies: not falling within the compass of their capacity: or where they are not made privy of the true and secret cause of such wonderful philosophical feats." These men are of four sorts, chiefly: the first I may name, vain prattling busybodies; the second, fond friends; the third, imperfectly zealous; and the fourth, malicious ignorant. To each of these (briefly and in charity) I will say a word or two, and so return to my Preface.

(1.)

Vain prattling busybodies: use your idle assemblies and conferences otherwise than in talk of matter either above your capacities for hardness, or contrary to your consciences in truth.

(2.)

Fond friends, leave off so to commend your unacquainted friend upon blind affection: as, because he knows more than the common student, that therefore he must necessarily be skillful and a doer in such matter and manner as you term conjuring. You think by that that you advance his fame, and that you make other men marvel greatly at your luck, to have such a learned friend. Cease to ascribe impiety where you pretend amity. For, if your tongues were true, then would that friend of yours be untrue both to God and his sovereign. Such friends and fondlings, I shake off and renounce you: shake off your folly.

(3.)

Imperfectly zealous, to you I say: that perhaps well you mean, but far you miss the mark: if a lamb you will kill to feed the flock with his blood. Sheep, with lamb's blood, have no natural sustenance: no more is Christ's flock duly edified with horrible slanders. Nor is your fair pretense any whit well graced by such rash ragged rhetoric. But such as so use me will find a foul crack in their credit. Speak what you know: and know as you ought: do not know by hearsay when life lies in danger. Search to the quick, and let charity be your guide.

(4.)

Malicious ignorant, what shall I say to you? Keep your tongue from evil. Refrain your tongue from slander. Though your tongues be sharpened serpent-like, and adders' poison lie in your lips:

(Psalm 140.)

yet take heed, and in time think to yourself: A man of loose tongue shall not be established in the land. Evil shall hunt the violent man until he is cast down. For I am sure that the Lord will do justice for the afflicted, and vengeance for the poor.

Thus I require you, my assured friends and countrymen (you mathematicians, mechanicians, and philosophers, charitable and discrete), to deal in my behalf with the frivolous and untrue-tongued, my envious adversaries or fond friends. And farther, I would wish that at leisure you would consider how Basilius Magnus lays Moses and Daniel before the eyes of those who count all such philosophical studies (as mine has been) ungodly or unprofitable. Weigh well S. Stephen's witness of Moses.

(Act. 7. C.)

Moses was instructed in all manner of wisdom of the Egyptians: and he was of power both in his words and works. You see this philosophical power and wisdom, which Moses had, to be in no way disapproved by the Holy Ghost. Yet Plinius has recorded Moses to be a wicked magician. And that of necessity must be either for this philosophical wisdom, learned before his calling to the leading of the children of Israel, or for those his wonders, wrought before King Pharaoh, after he had the conducting of the Israelites. As concerning the first, you per-

ceive how S. Stephen, at his martyrdom (being full of the Holy Ghost), in his recapitulation of the old testament, made mention of Moses' philosophy, with good liking of it: and Basilius Magnus also avouches it to have been profitable to Moses (and therefore, I say, to the church of God, necessary). But as concerning Moses' wonders, done before King Pharaos: God himself said: See that you do all those wonders before Pharaos, which I have put in your hand. Thus you evidently perceive how rashly Plinius has slandered Moses,

(Book 30, chapter 1.)

of vain fraudulent magic, saying: There is also another sect of magic, depending on Moses, Jannes, and Iotape, and the Jews, but many thousands of years after Zoroaster. etc.

(1.)

Let all such, therefore, who, in judgment and skill of philosophy, are far inferior to Plinie, "take good heed, for fear that they overshoot themselves rashly," in

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judging of philosophers' strange acts, and the means how they are done.

Thaumaturgike, 2



(2.)

But much more ought they to beware of forging, devising, and imagining monstrous feats and wonderful works when and where none such were done: not even a spark or likelihood of such as they report without any shame.

(3.)

And, to conclude, most of all let them be ashamed before men and afraid of the dreadful and just Judge: both for foolishly or maliciously devising such things, and then devilishly fathering their newly contrived monsters on me, though I am innocent in hand and heart, as though I had trespassed against the law of God or of man in any of my studies or exercises, philosophical or mathematical—which I hope will in due time be more manifest.

Archemastrie



Now I end, with Archemastrie. The name is not so new as the art is rare; for another art, a degree below this in skill and power, had already been given this English name. And yet this will serve our purpose well enough for now. This art teaches how to bring to actual sensible experience all worthy conclusions proposed by all the mathematical arts and concluded by true natural philosophy; and it both adds to them a further scope in the terms of the same arts, and also by its own method and in peculiar terms proceeds, with the help of the aforesaid arts, to the performance of complete experiences which of no particular art can formally be challenged. If you remember how we considered architecture in respect of all common handiworks, some light you may have from that toward understanding the sovereignty and property of this science. I may call it a science rather than an art, for the excellence and mastery it has over so many and so mighty arts and sciences. And because it proceeds by experiences, and searches out the causes of conclusions by experiences, and also puts the conclusions themselves into experience, it is named by some the Experimental Science. Nicolaus Cusanus calls it so in his Statical Experiments, and another philosopher, native to this land (the flower of whose worthy fame can never die nor wither), wrote at length on it at the request of Clement the Sixth. The art carries with it wonderful credit, for it certifies sensibly, fully, and completely, to

the utmost power of nature and art. This art certifies by complete and absolute experience; other arts, with their arguments and demonstrations, persuade, and in words prove their conclusions very well.

But words and arguments are no sensible certification, nor the full and final fruit of the sciences as they are practiced. And though some arts have experiences in them, yet they are not complete, nor brought to the utmost they may be stretched to and applied sensibly. As for example: the natural philosopher disputes and makes a goodly show of reason; and the astronomer and the optical mechanician put some things into experience—but neither all that they may, nor yet those they do, sufficiently and to the utmost. There then the Archemaster steps in, and leads the experiences on, by order of his experimental doctrine, to the chief and final power of natural and mathematical arts. Of two or three men in whom this description of Archemasonry was experimentally verified, I have read and heard, and there is good record of such perfection in them. So that this art is no fantastic imagination—as some sophister, with his Insolubles, might flourish, and dazzle your imagination, and dash your honest desire and courage from believing these things so unheard-of, so marvelous, and of such importance. Well: as you will. I have warned you. I have done the part of a friend: I have discharged my duty toward God, for my small talent received at his most merciful hands. To this science does the science Alnirangiat render great service. Do not wonder at this name: I do not change a name so used, and published in print by others, being a name proper to the science. Under this comes *Ars Sintrillia* [title unclear in the source], briefly written by Artephius. But the chief science of the Archemaster (in this world) as yet known is another (as it were) optical science: of which the name shall be told (God willing) when I shall have some more just occasion to discourse of it.

Here I must end, thus abruptly (gentle friend, and unfeigned lover of honest and necessary verities). For they who have, for your sake and virtue's cause, requested me (an old, worn-out mathematician) to take pen in hand (through the confidence they placed in my long experience and tried sincerity), for the declaring and reporting somewhat of the fruit and commodity to be attained by the mathematical arts: even they, sore against their wills, are forced, for

sundry causes, to satisfy the workman's request by ending without delay; he so fears this so new an attempt, and so costly, and in a matter so slenderly hitherto considered or esteemed among the common sort of students.

And since I was urged to set forth some reason why, in our vulgar speech, this part of the principal science of geometry, called Euclid's Geometrical Elements, is published for your handling—you who have no Latin, and are not university scholars—truly, I think it needless.

(1.)

For the honor and estimation of the universities and graduates is hereby nothing diminished, seeing that from them, and by those they have reared, you receive all this benefit, however great it be.

(2.)

Neither are their studies in the least hindered: no more than the Italian universities, as the academies of Bologna, Ferrara, Florence, Milan, Padua, Pavia, Perugia, Pisa, Rome, and Siena, or any one of them, find themselves at all disgraced, or their studies anything hindered, by Friar Lucas de Burgo or by Nicolaus Tartalea, who in the vulgar Italian language have published not only Euclid's Geometry, but somewhat of Archimedes: and in arithmetic and practical geometry very large volumes, all in their vulgar speech. Nor in Germany have the famous universities been anything discontent with Albertus Durerus, his geometrical institutions in German; or with Gulielmus Xylander, his learned translation of the first six books of Euclid out of the Greek into high German; nor with Gualterus H. Riffius, his geometrical volume, very diligently translated into the German tongue and published. Nor yet do the universities of Spain or Portugal think their reputation to be decayed, or suppose any of their studies hindered by the excellent P. Nonnius, his mathematical works put forth in vulgar speech. Have you not, likewise, in the French tongue the whole mathematical quadrivium? and yet have neither Paris, Orleans, nor any of the other universities of France ever been offended with the translators or publishers, or any man's study by that hindered?

(3.)

And surely the common and ordinary scholar (much more the grammarian) before coming to the university shall (or may) now be sufficiently instructed, according to Plato's counsel, in arithmetic and geometry, for the better and easier learning of all manner of philosophy, academic or peripatetic; and by that means go forward more cheerfully, more skillfully, and more speedily in his studies there to be learned; and so, in less time, profit more than otherwise he should or could do.

(4.)

Also many good and pregnant English wits, of young gentlemen and of others who never intend to meddle with the profound search and study of philosophy (to be learned in the universities), may nevertheless now, with more ease and liberty, have good occasion to occupy the sharpness of their wits virtuously: where otherwise, perhaps, they would spend their time—or rather lose it—in foolish exercises, neither serving God nor furthering the common or private good.

(5.)

And great comfort, with good hope, may the universities have, by reason of this English geometry and mathematical preface, that they hereafter shall be the more regarded, esteemed, and resorted to. For when it shall be known and reported that of the mathematical sciences only such great commodities are ensuing as I have specified, and that indeed some of you students with no Latin can be good witnesses of such rare fruit enjoyed by you by that, as either was not heard of before this, or else not so fully credited: "Well may all men conjecture that far greater aid and better furniture, to win to the perfection of all philosophy,

(Universities.)

may in the universities be had: being the storehouses and treasury of all sciences, and all arts necessary for the best and most noble state of commonwealths."

(6.)

Besides this, how many a common artisan is there in these realms of England and Ireland who deals with numbers, rule, and compass: who, with their own skill and experience already had, will be able (by these good helps and information) to find out and devise new works, strange engines, and instruments, for sundry purposes in the commonwealth? or for private pleasure? and for the better maintaining of their own estate? I will not therefore fight against my own shadow. For no man (I am sure) will open his mouth against this enterprise. No man (I say) who either has charity toward his brother (and would be glad of his furtherance in virtuous knowledge), or who has any care and zeal for the bettering of the common state of this realm. Neither any who make account of what the wiser sort of men (sage and steady) think of them. To none, therefore, will I make any apology for doing a virtuous act, and for commending or setting forth profitable arts to English men in the English tongue. "But to God our Creator let us all be thankful: for that, as he, of his goodness, by his power, and in his wisdom, has created all things in number, weight, and measure: so, to us, of his great mercy, he has revealed means by which to attain the sufficient and necessary knowledge of his three principal instruments afore-said: which means I have abundantly proved to you to be the sciences and arts mathematical."

And though I have been pinched with straightness of time, that in no way I could pen down the matter (in my mind) as I determined, hoping for convenient leisure: yet if virtuous zeal and honest intent provoke and bring you to the reading and examining of this compendious treatise, I do not doubt but as the truth of it (according to our purpose) will be evident to you, so the pith and force of it will persuade you, and the wonderful fruit of it highly please you. And that you may the easier perceive, and better remember, the principal points of which my preface treats,

(The ground plan of this preface in a table.)

I will give you the ground plan of my whole discourse, in a table annexed, from the first to the last, somewhat methodically contrived.

If haste has caused my poor pen anywhere to stumble, you will (I am sure), in partial recompense for my earnest and sincere good will in pleasing you, consider the huge rocky mountains and the perilous unbeaten ways through which it (both night and day the while) has toiled and labored to bring you this good news and comfortable proof of virtue's fruit.

So I commit you to God's merciful direction for the rest: heartily beseeching him to prosper your studies and honest intentions, to his glory and the benefit of our country. Amen.

Written at my poor house at Mortlake.

In the year 1570. February 9.

J. Dee

The Groundplat of the Mathematical Praeface



Here you have (as I promised) the ground plan of my Mathematical Preface, annexed to Euclid and now first published in our English tongue. An. 1570. Febr. 3.

Sciences and arts mathematical are either

Principal, which are two only,

Arithmetic.

Simple, which deals with numbers only, and demonstrates all their properties and appurtenances, where a unit is indivisible. Mixed, which, with the help of principal geometry, demonstrates some arithmetical conclusion or purpose.

Geometry.

Simple, which deals with magnitudes only, and demonstrates all their properties, affections, and appurtenances, whose point is indivisible. Mixed, which, with the help of principal arithmetic, demonstrates some geometrical purpose, as Euclid's Elements.

The use of these is either:

In supernatural, eternal, and divine things: by application, ascending. In mathematical things: without further application. In natural things, both substantial and accidental, visible and invisible, etc.: by application, descending.

The like uses and applications are found, though a degree lower, in the derivative mathematical arts.

Derivative from the principal ones: of which some have

The names of the principal ones, as:

Arithmetic, vulgar, which considers

—Arithmetic of the most usual whole numbers, and of the fractions belonging to them.—Arithmetic of proportions.—Circular arithmetic.—Arithmetic of radical numbers: simple, compound, and mixed; and of their fractions.—Arithmetic of cossic numbers, with their fractions; and the great art of algebra.

Geometry, vulgar, which teaches measuring

At hand

All lengths—Mecometrie. All planes, such as land, board, glass, etc.—Embadometrie. All solids, such as timber, stone, vessels, etc.—Stereometrie.

With distance from the thing measured, as:

How far any thing is from the measurer, seen by him on land or water, is called Apomecometrie. How high or deep any thing is from the level of the measurer's standing, seen by him on land or water, is called Hypsometrie. How broad a thing is that lies in the measurer's view, so it be situated on land or water, is called Platometrie.

From which have grown the feats and arts of

Geodesy, more skillfully to measure and survey lands, woods, waters, etc. Geography. Chorography. Hydrography. Stratarithmetrie.

Proper names, as:

Perspective—which demonstrates the modes and properties of all radiations: direct, broken, and reflected.

Astronomy—which demonstrates the distances, magnitudes, and all natural motions, appearances, and affections proper to the planets and fixed stars: for any time, past, present, and to come: in respect of a certain horizon, or without respect of any horizon.

Music—which demonstrates by reason, and teaches by sense, perfectly to judge and order the diversity of sounds, high or low.

Cosmography—which wholly and perfectly makes description of the heavenly and also the elemental part of the world: and of these parts makes homologous application and mutual collation necessary.

Astrology—which reasonably demonstrates the operations and effects of the natural beams of light and the secret influence of the planets and fixed stars in every element and elemental body: at all times, in any horizon assigned.

Statics—which demonstrates the causes of heaviness and lightness of all things: and of the motions and properties belonging to heaviness and lightness.

Anthropography—which describes the number, measure, weight, figure, situation, and color of every diverse thing contained in the perfect body of man: and gives certain knowledge of the figure, symmetry, weight, characterization, and due local motion of any designated part of the said body, and of the numbers belonging to that part.

Trochilike—which demonstrates the properties of all circular motions: simple and compound.

Helicosophy—which demonstrates the designing of all spiral lines in a plane, on a cylinder, cone, sphere, conoid, and spheroid, and their properties.

Pneumatithmie—which demonstrates by closed hollow geometric figures (regular and irregular) the strange properties (in motion or rest) of water, air, smoke, and fire in their continuity, and as they are joined to the elements next to them.

Menadrie—which demonstrates how virtue and force may be multiplied above nature's simple virtue and power: and so to direct, to lift, to pull toward, and to put or cast from, any determined virtue, weight, or force, simple or multiplied, which naturally cannot be so directed or moved.

Hypogeiodie—which demonstrates how, under the spherical surface of the earth, at any depth, to any assigned perpendicular line (whose distance from the perpendicular of the entrance, and the azimuth likewise in respect of the said entrance, being known), a certain course may be prescribed and traveled, etc.

Hydragogie—which demonstrates the possible leading of water by nature's law and by artificial help from any head (being spring, standing, or running water) to any other place assigned.

Horometry—which demonstrates how, at all times appointed, the precise customary designation of time may be known for any place assigned.

Painting—which demonstrates and teaches how the intersection of all visual pyramids made by any assigned plane (the center, distance, and lights being determined) may be represented by lines and proper colors.

Architecture—which is a science adorned with many doctrines and diverse instructions: by whose judgment all works finished by other workmen are judged.

Navigation—which demonstrates how, by the shortest good way, by the aptest direction, and in the shortest time, a sufficient ship between any two places (in a navigable passage) assigned may be conducted: and in all storms and natural disturbances happening, how to use the best possible means to recover the place first assigned.

Thaumaturgike—which gives certain order to make strange works, to be perceived by the senses, and greatly to be wondered at by men.

Archemastrie—which teaches how to bring to actual sensible experience all worthy conclusions proposed by all the mathematical arts and concluded by true natural philosophy: and both adds to them a further scope in the terms of the same arts, and also by its proper method and in peculiar terms proceeds, with the help of the aforesaid arts, to the performance of complete experiences which of no particular art can formally be challenged.

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